HW6, due Tuesday, March 3:
7616 #1-5
7626 #1.2
7632 #1
7634 #1
7638 #1.2

Today:

- Second-order DE with constant coefficients
- Mass spring problems

Example 0. Recall how to find the general solution to this DE:
\[ y' - 4y = \cos t. \]

This is an example of a non-homogeneous first-order linear differential equation with constant coefficients.

Recall that the general solution is the sum of a particular solution to this equation and the general solution of the associated homogeneous DE \( y' - 4y = 0 \).

Example 1. Find the general solution:
\[ 2y'' - 5y' - 3y = 0 \]

This is an example of a homogeneous second-order linear differential equation with constant coefficients.

Example 2. Solve the initial value problem:
\[ y'' - 6y' + 9y = 0, \]
\[ y(0) = 1, \]
\[ y'(0) = 2. \]

Rule: if the auxiliary equation has a repeated root \( m \), then \( e^{mt} \) and \( te^{mt} \) are both solutions.

Example 3. Find the general solution:
\[ y'' - 6y' + 25y = 0. \]
**Summary:** Associated to each second-order homogeneous DE with constant coefficients is an *auxiliary equation*.

- If the auxiliary equation has distinct real roots $m_1$ and $m_2$, then the general solution is $C_1e^{m_1t} + C_2e^{m_2t}$.

- If the auxiliary equation has a repeated real root $m$, then the general solution is $C_1e^{mt} + C_2te^{mt}$.

- If the auxiliary equation has complex roots $a \pm bi$, then the general solution is $C_1e^{at}\cos(bt) + C_2e^{at}\sin(bt)$.

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**Spring-mass problems**

**Example 4.** Suppose that a mass of 1 kg is hanging from a spring with spring constant 4 kg/sec\(^2\). Suppose that there is no external force acting on the spring. Suppose that the spring is pulled to a point 0.1 meters below equilibrium and at $t = 0$ is let go. Find an expression for the movement of the spring under the following assumptions:

a. there is no damping force (vacuum)

b. the damping constant is 2 kg/sec

c. the damping constant is 4 kg/sec

d. the damping constant is 8 kg/sec

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**Summary:** If there is no external force, then a spring-mass system leads to a homogeneous second-order DE with constant coefficients.

- If the auxiliary equation has distinct real roots $m_1$ and $m_2$, then we say the system is *overdamped*.

- If the auxiliary equation has a repeated real root $m$, then we say the system is *critically damped*.

- If the auxiliary equation has complex roots $a \pm bi$, then we say the system is *underdamped*.