MATH 2373.020 – Lecture 10 – 2/23/09

HW6, due Tuesday, March 3:
7616 #1-5
7626 #1,2
7632 #1
7634 #1
7638 #1,2

Today:

- Second-order DE with constant coefficients
- mass spring problems

Example 0. Recall how to find the general solution to this DE:

$$y' - 4y = \cos t.$$

This is an example of a non-homogeneous first-order linear differential equation with constant coefficients.

Recall that the general solution is the sum of a particular solution to this equation and the general solution of the associated homogeneous DE $y' - 4y = 0$.

Example 1. Find the general solution:

$$2y'' - 5y' - 3y = 0$$

This is an example of a homogeneous second-order linear differential equation with constant coefficients.

Example 2. Solve the initial value problem:

$$y'' - 6y' + 9y = 0,$$

$$y(0) = 1,$$

$$y'(0) = 2.$$

Rule: if the auxiliary equation has a repeated root m, then e^{mt} and te^{mt} are both solutions.

Example 3. Find the general solution:

$$y'' - 6y' + 25y = 0.$$

Summary: Associated to each second-order homogeneous DE with constant coefficients is an *auxiliary equation*.

- If the auxiliary equation has distinct real roots m_1 and m_2, then the general solution is $C_1e^{m_1t} + C_2e^{m_2t}$.

- If the auxiliary equation has a repeated real root m, then the general solution is $C_1e^{mt} + C_2te^{mt}$.

- If the auxiliary equation has complex roots $a \pm bi$, then the general solution is $C_1e^{at}\cos(bt) + C_2e^{at}\sin(bt)$.

Summary: If there is no external force, then a spring-mass system leads to a homogeneous second-order DE with constant coefficients.

- If the auxiliary equation has distinct real roots m_1 and m_2, then we say the system is *overdamped*.

- If the auxiliary equation has a repeated real root m, then we say the system is *critically damped*.

- If the auxiliary equation has complex roots $a \pm bi$, then we say the system is *underdamped*.

Spring-mass problems

Example 4. Suppose that a mass of 1 kg is hanging from a spring with spring constant 4 kg/sec^2. Suppose that there is no external force acting on the spring. Suppose that the spring is pulled to a point 0.1 meters below equilibrium and at $t = 0$ is let go. Find an expression for the movement of the spring under the following assumptions:

a. there is no damping force (vacuum)

b. the damping constant is 2 kg/sec

c. the damping constant is 4 kg/sec

d. the damping constant is 8 kg/sec