To make sense of the concept of eigenvalue, let’s think of $n \times n$ matrices $A$ as mapping $n$-dimensional vectors to other $n$-dimensional vectors.

Examples when $n = 2$:

**Example a.**

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

**Example b.**

$$B = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

If $A$ is an $n \times n$ matrix, then the equation

$$Av = \lambda v$$

has nontrivial solutions $v$ if and only if

$$\det(A - \lambda I) = 0.$$ 

The expression $\det(A - \lambda I)$ is called the characteristic polynomial of $A$. Its roots are the eigenvalues of $A$.

**Example 2.** Find the eigenvalues, and corresponding eigenvectors, for the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}.$$
Example 3. Find the eigenvalues, and corresponding eigenvectors, for the matrix

\[
A = \begin{bmatrix}
1 & 1 & 0 \\
1 & 1 & 1 \\
0 & 1 & 1
\end{bmatrix}.
\]

What will this have to do with DE?