MATH 2373.020 – Lecture 13 – 3/4/09

HW7, due Tuesday, March 10:
7620 #1,2
7626 #4-6
7656 #1-5
7632 #2-4
7634 #2-5
7640 #1-5

Today:

- eigenvalues and eigenvectors

To make sense of the concept of eigenvalue, let's think of $n \times n$ matrices A as mapping n-dimensional vectors to other n-dimensional vectors.

Examples when $n = 2$:

Example a.

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Example b.

$$B = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

Eigenvalues and eigenvectors

If A is an $n \times n$ matrix, and v is an n-dimensional vector, then Av is also an n-dimensional vector.

We ask: is there a number λ so that

$$Av = \lambda v$$

for some nonzero vector v?

If so, we say that λ is an *eigenvalue* of A and v is an *eigenvector* of A.

Example 1. Find the eigenvalues, and corresponding eigenvectors, for the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix}.$$

If A is an $n \times n$ matrix, then the equation

$$Av = \lambda v$$

has nontrivial solutions v is and only if

$$\det(A - \lambda I) = 0.$$

The expression $\det(A - \lambda I)$ is called the *characteristic polynomial* of A. Its roots are the eigenvalues of A.

Example 2. Find the eigenvalues, and corresponding eigenvectors, for the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}.$$

Now go back to examples a and b:

Example a.

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Example b.

$$B = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

Example 3. Find the eigenvalues, and corresponding eigenvectors, for the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

What will this have to do with DE?