Today:
- reduction of order
- second-order non-homogeneous with non-constant coefficients
- complex eigenvalues and -vectors

Here we have a second-order non-homogeneous DE with constant coefficients AND the coefficient on \( y \) is zero.

**Example 1.** Find the general solution of the differential equation

\[ y'' + 2y' = e^{2t}. \]

The method can be applied just as well to the constant coefficient case, with benefits:

**Example 3.** Find the general solution of the differential equation

\[ y'' - 6y' + 9y = 0. \]

Note. We have already solved equations of this type, but it required an assumption: that if the auxiliary equation has a repeated root \( m \), then not only \( e^{mx} \) BUT ALSO \( xe^{mx} \) are solutions. We can now justify that assumption.

**Example 4.** Find the general solution of the differential equation

\[ y'' - 2y' - 3y = e^{4t}. \]

In some cases, as in example 1, reduction of order takes the place of using the method of undetermined coefficients on non-homogeneous DE:

**Example 2.** Find the general solution of the differential equation

\[ x^2y'' + 2xy' - 6y = 0, \]
given that \( y_1(x) = x^2 \) is a solution.

HW8, due Tuesday, March 24:
7620 #3-6
7656 #6-10
7662 #1-4
7636 #1-5
7638 #3,4
7641 #1,2

HW9, due Thursday, March 26:
7620 #7
7668 #1-5
7646 #1-3

Exam 2: Thursday, March 26

Here we have a second-order homogeneous DE with non-constant coefficients. Generally these are difficult to solve, but if we know one solution, then it is straightforward to find another, linearly independent solution.

**Example 2.** Find the general solution of the differential equation

\[ x^2y'' + 2xy' - 6y = 0, \]
given that \( y_1(x) = x^2 \) is a solution.
Why does this work?

That is, why does the substitution $y_2 = y_1 u$ always lead to a new DE where the order can be reduced?

Back to eigenvalues:

**Example 5.** Find eigenvalues and corresponding eigenvectors for the matrix

\[
\begin{pmatrix}
  -2 & 13 \\
  -1 & 4 \\
\end{pmatrix}
\]