MATH 2373.020 – Lecture 18 – 3/30/09

HW10, due Tuesday, April 7:
7703 #1-4
7705 #1,2
7725 #1-6
7756 #1,2,4
7762 #1,2,4-7
7776 #1,2
7778 #1,2

Office hour Wed. after class, not at 10:30

Today:

- eigenvalues and systems of first-order differential equations
- double eigenvalues and their eigenvectors
- "reducing order" using systems of first-order DE
- Introduction to the Laplace transform

Example 1. Find the general solution of the system of differential equations

$$\frac{dx}{dt} = 2x + 3y$$

$$\frac{dy}{dt} = 2x + y.$$

Summary: when coefficient matrix has two different real eigenvalues

The general solution of the system $X' = AX$ where $X = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ is

$$X = c_1 V e^{\lambda_1 t} + c_2 W e^{\lambda_2 t},$$

where V and W are eigenvectors corresponding, respectively, to the different real eigenvalues λ_1 and λ_2 of the matrix A.

Example 2. Solve the initial value problem

$$x' = x + 2y$$

$$y' = 4x + 3y,$$

with initial conditions $x(0) = 1$ and $y(0) = 8$.

We will consider later the cases where the coefficient matrix of the system has

- complex eigenvalues
- a double eigenvalue

For now, let's work (for the first time) with a matrix that has a double eigenvalue:

Example 3. Find eigenvalues and eigenvectors for the matrix

$$\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}.$$

Example $3\frac{1}{2}$. Note: a double eigenvalue is not guaranteed to have two independent eigenvectors. Consider

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}.$$

Example 4. How did we solve this second-order differential equation?

$$y'' + 7y' + 12y = 0$$

Rewrite as a first-order system to produce another approach to solving.

The Laplace transform

The Laplace transform

1. **What is a *transform*?**

An operation that changes one function into another.

2. **Why do we need it?**

We've worked with second-order non-homogeneous DE like this:

$$ay'' + by' + cy = f(t).$$

The function $f(t)$ may not even be continuous. Laplace transforms help us solve problems of that type.

The Laplace transform

Given a function $f(t)$ defined for $t \geq 0$, the *Laplace transform* of $f(t)$ is defined to be

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt,$$

when the integral exists.

Example 5. Let

$$f(t) = \begin{cases} 2 & \text{for } 3 \leq t \leq 6; \\ 0 & \text{for } 0 \leq t < 3 \text{ and for } t > 6. \end{cases}$$

Find $\mathcal{L}\{f(t)\}$.

Try:

$\mathcal{L}\{C\}$, that is, $f(t) = C$, a constant.

$\mathcal{L}\{t\}$

$\mathcal{L}\{\sin t\}$