HW10, due tomorrow:
7703 #1-4
7705 #1,2
7725 #1-6
7756 #1,2,4
7762 #1,2,4-7
7776 #1,2
7778 #1,2

Today:

- 2x2 systems with complex eigenvalues
- inverse Laplace transforms
- “standard form” for non-homogeneous first-order systems

A common theme here is “algebra prep” – getting expressions into a form so that we can apply our existing theory.

Example 1. Solve the system:
\[
\begin{align*}
x' &= x - 2y, \\
y' &= 2x + y.
\end{align*}
\]

Summary: Given a 2x2 system \( X' = MX \), if \( M \) has complex eigenvalues, then two linearly independent solutions are the real and imaginary parts of one of the complex solutions.

Recall: a table of Laplace transforms
\[
\begin{align*}
\mathcal{L}\{1\} &= \frac{1}{s} \\
\mathcal{L}\{t^n\} &= \frac{n!}{s^{n+1}} \\
\mathcal{L}\{e^{at}\} &= \frac{1}{s-a} \\
\mathcal{L}\{\cos bt\} &= \frac{s}{s^2 + b^2} \\
\mathcal{L}\{\sin bt\} &= \frac{b}{s^2 + b^2}
\end{align*}
\]

Now: the inverse Laplace transform
\[
\mathcal{L}^{-1}\{F(s)\} \text{ is defined to be the function } f(t) \text{ such that } \mathcal{L}\{f(t)\} = F(s).
\]

Theorem. \( \mathcal{L}^{-1} \) is linear, just like \( \mathcal{L} \).

Example 2. Find \( \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} \).

Example 3. Find \( \mathcal{L}^{-1}\left\{\frac{5}{s^2}\right\} \).

Example 4. Find \( \mathcal{L}^{-1}\left\{\frac{e^{t^2}}{s^2+1}\right\} \).

Example 5. Find \( \mathcal{L}^{-1}\left\{\frac{e^{t^2}}{s^2+1}\right\} \).

Example 6. Find \( \mathcal{L}^{-1}\{3\} \).
We have written homogeneous systems in the form $\mathbf{X}' = M \mathbf{X}$.

**Example 7.** Write the following system in the form $\mathbf{X}' = B \mathbf{X} + \mathbf{D}$, where $\mathbf{D}$ is a vector of functions of $t$:

\[
\begin{align*}
x' + 2y' + 3x + 4y &= 3e^{2t} \\
2x' - y' - x + 2y &= 2e^{3t}.
\end{align*}
\]

Goal: Write an equivalent system where one equation involves $x'$ and not $y'$, and the other involves $y'$ and not $x'$. 