HW11, due tomorrow:
7707 #1-8
7709 #1-3
7727 #1-4
7762 #8-10
7766 #1-6
7778 #3

Today:

• 2x2 non-homogeneous systems
• more on the first translation theorem
• the second translation theorem
• more inverse Laplace transforms
• ... and why we need them???

Example 1. Find the general solution of the following non-homogeneous system:
\[
\begin{align*}
x' &= x - y + 3e^t \\
y' &= x + y + 3e^t.
\end{align*}
\]

Example 2. Find the general solution of the following non-homogeneous system:
\[
\begin{align*}
x' &= 6x + y + 6t \\
y' &= 4x + 3y - 10t + 4.
\end{align*}
\]

More on the first translation theorem

“A factor of \(e^{at}\) shifts the Laplace transform to the right \(a\) units.”

Example 3. Find \(L\{e^{-at}\}\).

How do we deal with a factor of \(t\) generally, like in \(L\{t \sin 5t\}\)?

Theorem. \(L\{tf(t)\} = -F'(s)\).

The second translation theorem

Now we will see how to handle functions involving \(H(t-c)\):

Theorem. \(L\{f(t-c)H(t-c)\} = e^{-cs}L\{f(t)\}\).

Example 4. Find \(L\{\sin(t - \frac{\pi}{2})H(t - \frac{\pi}{2})\}\).
The problem is, most products we encounter will be of the form $f(t)H(t-c)$. We have to rewrite $f(t)$ as a shift of some function: $f(t) = g(t-c)$.

**Example 5.** Find $\mathcal{L}((t^2 + 3t - 1)H(t-2))$.

We can more simply use an alternate form of the second translation theorem: writing $f(t) = g(t-c)$, we have

$$\mathcal{L}\{f(t)H(t-c)\} = \mathcal{L}\{g(t-c)H(t-c)\} = e^{-cs}\mathcal{L}\{g(t)\} = e^{-cs}\mathcal{L}\{f(t+c)\}.$$  

$$\mathcal{L}\{f(t)H(t-c)\} = e^{-cs}\mathcal{L}\{f(t+c)\}.$$  

Redo Example 5.

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The table:

- $\mathcal{L}\{1\} = \frac{1}{s}$
- $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$
- $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$
- $\mathcal{L}\{\cos bt\} = \frac{s}{s^2 + b^2}$
- $\mathcal{L}\{\sin bt\} = \frac{b}{s^2 + b^2}$

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More partial fractions... in order to find inverse Laplace transforms

**Example 6.** Find

$$\mathcal{L}^{-1}\left\{\frac{6s^2 + 50}{(s + 3)(s^2 + 4)}\right\}.$$

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How do these inverse Laplace transforms come up?

**Example 7.** Solve the initial value problem

$$y'' + 3y = 13\sin 2t,$$

$$y(0) = 6.$$