MATH 2373.020 – Lecture 26 – 4/27/09

HW13, due tomorrow:
7715 #2, 3
7731 #1-5
7733 #1
7735 #1
7774 #1-5
7778 #4, 5, 6

HW14, due Thursday, 4/30:
7715 #4, 5
7733 #2, 3, 4
7784 #1, 2, 3

Exam 3, Thursday, 4/30: Physics 150.
Final Exam, Monday, May 11, 1:30 - 4:30

Today:
- begin review for Exam III
- second-order using Laplace
- three-spring, two-mass system
- two-tank system?

Today:
The table that will be given to you on the test:

\[
\begin{align*}
\mathcal{L} \left( a^n t^n \right) & = \frac{a^n}{s^n} \\
\mathcal{L} \left( e^{at} \right) & = \frac{1}{s-a} \\
\mathcal{L} \left( \cos bt \right) & = \frac{s}{s^2 + b^2} \\
\mathcal{L} \left( \sin bt \right) & = \frac{b}{s^2 + b^2} \\
\mathcal{L} \left( f'(t) \right) & = sF(s) - f(0) \\
\mathcal{L} \left( f''(t) \right) & = s^2F(s) - sf(0) - f'(0) \\
\mathcal{L} \left( e^{at} f(t) \right) & = F(s-a) \text{ (first translation theorem)} \\
\mathcal{L} \left( f(t-c)H(t-c) \right) & = e^{-cs}F(s) \text{ (second translation theorem)}
\end{align*}
\]

Example 1. Solve the initial value problem

\[
x'' - 9x = 2 + 5H(t - 2) - 7H(t - 5),
\]

\[
x(0) = 1,
\]

\[
x'(0) = -3.
\]

Example 2. Write an initial-value system that models the following situation:

- A mass of 1 kg is attached to a mass of 2 kg by a spring with constant \(k_{12} = 6\).
- The 1 kg mass is attached to a wall on the left by a spring with constant \(k_1 = 3\).
- The 2 kg mass is attached to a wall on the right by a spring with constant \(k_2 = 4\).
- The system is set in motion pulling the left mass to the left 1 m and the right mass to the right 1 m, and then letting them go.

Then, reduce the order of the system and “solve” it.

Example 3. (from 4/15) Write a system of differential equations that models the quantities of salt in the two tanks in the following scenario:

Initially, tank A contains 5 lb of salt dissolved in 100 gal of water, and tank B contains 10 lb of salt dissolved in 150 gal of water.

Brine is flowing as follows:

1. into tank A, 3 gal/min, 2 lb/gal
2. into tank B, 4 gal/min, 3 lb/gal
3. from A to B, 1 gal/min
4. from B to A, 2 gal/min
5. draining from A, 4 gal/min
6. draining from B, 3 gal/min