Today:
- begin to review for the final exam
- differential equations involving the delta function
- systems with one eigenvector
- feedback

Wednesday:
- final exam review. Send me email after you receive the (5) review sheets Tuesday to suggest review problems.

Example 1. Solve the initial value problem
\[ y' - 5y = e^{-4(t-3)}H(t-3), \]
\[ y(0) = 2. \]

Recall: The delta function is defined to be
\[ \delta(t) = \lim_{h \to 0} g_h(t), \]
where \( g_h(t) = \frac{1}{h} \) on the interval \([0, h]\). We showed that
\[ \mathcal{L}\{\delta(t - c)\} = e^{-cs}. \]

Example 2. Solve the initial value problem
\[ x'' - x' - 6x = 5\delta(t - 2), \]
\[ x(0) = 1, \]
\[ x'(0) = 2. \]

What to do when the coefficient matrix has a double eigenvalue.

Example 3. Find the general solution of the system \( X' = AX \), where
\[ A = \begin{pmatrix} 5 & -2 \\ 2 & 1 \end{pmatrix}. \]
If the coefficient matrix $A$ has a double eigenvalue $\lambda$ with corresponding eigenvector $V$, then one solution is

$$X_1 = Ve^\lambda,$$

and a second linearly independent solution is

$$X_2 = Vet e^\lambda + We^\lambda,$$

where $W$ is a solution to the linear system

$$(A - \lambda I)W = V.$$

**TIP:** To find a vector for $e^\lambda$ in the second solution, look at the augmented matrix $(A - \lambda I|V)$.

**Example 4.** Find the general solution of the system $X' = AX$, where

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}.$$