This exams contains 5 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one (doubled-sided) 8.5 inch × 11 inch sheet of notes into the exams.

Do not give numerical approximations to quantities such as \( \sin 5 \), \( \pi \), or \( \sqrt{2} \). However, you should simplify \( \cos \frac{\pi}{2} = 0 \), \( e^0 = 1 \), and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals.**

- **Mysterious or unsupported answers will not receive full credit.** Your work should be mathematically correct and carefully and legibly written.

- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit;** an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive very little credit.

Useful formulas

\[
\begin{align*}
\sin 2x &= 2 \sin x \cos x \\
\cos 2x &= \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x \\
\tan 2x &= \frac{2 \tan x}{1 - \tan^2 x}
\end{align*}
\]

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1. (30 points) Calculate \( \int \int_W \sqrt{x^2 + y^2} \sqrt{x^2 + y^2 + z^2} \, dx \, dy \, dz \) where \( W \) is the quarter sphere defined by \( x^2 + y^2 + z^2 \leq 9 \), \( y \geq 0 \), \( z \geq 0 \).
2. (35 points) Compute the integral

\[ \int \int_D \sqrt[3]{\frac{y^2}{x}} \, dA \]

over the region \( D \) defined by \( 0 \leq x^2/y \leq 1 \) and \( 0 \leq y^2/x \leq 8 \), using the change of variables \( u = \sqrt[3]{y^2/x} \) and \( v = \sqrt[3]{x^2/y} \). You may use the fact that, under this change of variables, \( x = uv^2 \) and \( y = u^2v \).
3. (45 points) Let \( F(x, y, z) = (xy, e^{z^2} + y, x + y) \), and let the surface \( S \) be the graph of the function \( y = \frac{x^2}{9} + \frac{z^2}{9} - 1 \) with \( y \leq 0 \), oriented so that the normal vector to \( S \) has positive \( y \) component.

(i) (30 points) Using Stokes’ Theorem, convert the integral \( \int_S \nabla \times F \cdot d\mathbf{S} \) to a line integral.

(ii) (15 points) Evaluate the line integral.
4. (40 points) Let $G(x, y, z) = (1 - 2z e^z, -1, -x)$, and let the surface $S$ be the disc of radius 3 in the $xz$-plane, centered at the origin, with normal vector pointing in the direction of $(0, -1, 0)$.

(i) (30 points) Compute the integral $\iiint_S G \cdot dS$.

(ii) (10 points) Explain how your answer relates to that of question 3. Be sure to include in your response how any theorems justify your explanation.