MATH 2374.020 – Lecture 10 – 2/22/08

Place HW 04 in folders in back center of the room.

Reminder: Review session here Monday 1:25. Your questions will guide the discussion. Also, extra office hours Monday 2:15 - 4.

HW 05, due Fri. 2/29: 3.1 #6, 9, 15b, 16; 5.1 #4, 6, 7; 5.2 #2c, 4, 6; 5.3 #2a, 6, 9.

Quiz 11: due Fri. 2/29, 9 a.m. [This has been corrected since class. See also the Announcement on the WebVista site.]

Exam 1: 2/27, 5 or 6:10, 375 SciCB. Additional information on the course website.

Today:

(5.3) The double integral over more general regions than rectangles.

Example 3 from Wed. Find
\[
\int \int_R xye^{-(x^2+y^2)} \, dA,
\]
where \( R = [0, A] \times [0, A] \) for fixed \( A > 0 \). How does this quantity behave as \( A \to \infty \)?

Example 1. Find the volume of the solid that lies beneath the graph of \( z = x^2 \) and over the region in the \( xy \)-plane that is bounded by the curves \( y = 4 - x^2 \) and \( y = x^2 - 4 \).

Example 2. Find the volume of the solid that lies beneath the graph of \( z = 1 \) and over the region in the \( xy \)-plane that is bounded by the curves \( x = .1 \sin 5y, \ x = 1 + .1 \sin 5y, \ y = 0, \) and \( y = 2\pi \).

Example 2’. Set up the double integral that equals the volume of the solid that lies between the graphs of \( z = .1 \sin y \) and \( z = 1+.1 \sin y \) and over the region in the \( xy \)-plane that is bounded by the curves \( x = .1 \sin 5y, \ x = 1+.1 \sin 5y, \ y = 0, \) and \( y = 2\pi \).
By finding the volume of the solid in example 2, we also found the area of the region in the $xy$-plane.

**Principle:** To find the area $A(R)$ of a region $R$ in the $xy$-plane, integrate the function $z = 1$ over that region. That is,

$$A(R) = \iint_R 1 \, dA$$

In example 2', we were using the fact that the volume of the solid that lies between two graphs $f(x,y) \leq g(x,y)$ over a region $R$ is the following:

$$V = \iint_R g(x,y) \, dA - \iint_R f(x,y) \, dA.$$  

**Example 3.** Estimate the value of

$$\iint_R e^{\sin x \cos y} \, dA,$$

where $R$ is the region in the $xy$-plane bounded by the graph of the equation $x^2 + y^2 = 4\pi^2$. 