MATH 2374.020 – Lecture 13 – 3/5/08

HW 06, due Fri. 3/7: 5.4 #2b, 11, 12; 5.5 #7, 10, 15, 20.
Quiz 13: due Fri. 3/7, 9 a.m.
Quiz 14: due Wed. 3/12, 9 a.m.
Lab 4a is due the week of 3/24, with Lab 4b.

Exam 1 statistics
205 students, 140 points possible
median 91, mean 90
Suggested grades:

<table>
<thead>
<tr>
<th>grade</th>
<th>range</th>
<th>number of students</th>
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<tr>
<td>A</td>
<td>[110, 140)</td>
<td>28</td>
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<tr>
<td>A-</td>
<td>[103, 109]</td>
<td>26</td>
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<tr>
<td>B+</td>
<td>[97, 102]</td>
<td>22</td>
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<tr>
<td>B</td>
<td>[94, 96]</td>
<td>19</td>
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<tr>
<td>B-</td>
<td>[88, 93]</td>
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<td>C+</td>
<td>[83, 87]</td>
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<tr>
<td>C</td>
<td>[77, 82]</td>
<td>23</td>
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<tr>
<td>C-</td>
<td>[67, 76]</td>
<td>18</td>
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Today:

Paths:
1. operations on paths
2. velocity and acceleration of paths; Newton’s second law
3. length of paths

Product rules

Let \( c_1, c_2 : \mathbb{R} \to \mathbb{R}^3 \) be paths in \( \mathbb{R}^3 \).

Then \( c_1 \times c_2 \) is also a path in \( \mathbb{R}^3 \).

Also, \( c_1 \cdot c_2 \) is a real-valued function.

How do we find their derivatives? Via the product rule:

\[
\frac{d}{dt}[c_1(t) \times c_2(t)] = c_1'(t) \times c_2(t) + c_1(t) \times c_2'(t).
\]

\[
\frac{d}{dt}[c_1(t) \cdot c_2(t)] = c_1'(t) \cdot c_2(t) + c_1(t) \cdot c_2'(t).
\]

Example 1. Let \( c_1(t) = (\cos t, \sin t, t) \) and let \( c_2(t) = (1, t, t^2) \).
Find the derivatives of the cross product and the dot product of \( c_1 \) and \( c_2 \) directly and via the product rules.

Velocity, acceleration, and Newton’s second law

The velocity of a path \( c \) is given by \( c' \), and its acceleration is given by \( a = c'' \).

If a particle following path \( c \) has mass \( m \), then the force acting on the particle to create acceleration \( a \) is given by

\[
F(c(t)) = ma(t).
\]
Example 2. Let \( c(t) = (r \cos kt, r \sin kt), \, r, k > 0 \). This is a circular path with radius \( r \) that completes one revolution every \( 2\pi/k \) seconds.

Find the speed of the particle.

Find the force acting on a 1 kg particle that keeps it in this orbit.

Example 3. Find the centripetal force acting on a mass of 4 kg that is moving in a circular path of radius 10 meters with frequency 2 revolutions per second.

Arc length

If \( c(t) = (x(t), y(t), z(t)) \) is a path in \( \mathbb{R}^3 \), then the length of the path from \( t_0 \) to \( t_1 \) is

\[
L = \int_{t_0}^{t_1} \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} \, dt
\]

\[
= \int_{t_0}^{t_1} ||c'(t)|| \, dt.
\]

Example 4. Find the lengths of the paths \( c_1(t) = (\cos t, \sin t, t) \) and \( c_2(t) = (\cos t, \sin t, t^2) \) from \( t_0 = 0 \) and \( t_1 = 1 \).