MATH 2374.020 – Lecture 18 – 3/28/08

Place HW 08 in folders in back center of the room.

HW 09: due Fri. 4/4. 8.3 #4, 7, 13ac, 15a, 59b.

Quiz 17: due Wed. 4/2.

Exam 2: Wed. 4/2, 5 or 6 p.m. Will cover 3.1, 5.1 - 5.5, 4.1 - 4.4, 7.1, 7.2, 8.1.

Review session: Mon. 3/31, 1:25 - 2:15 here. Bring questions from old exams, HW, etc.

Additional office hours: Mon. 3/31, 2:15 - 4.

Today:

After the detail of Wednesday, let’s step back and ask, what is Green’s Theorem good for? And work some examples, as a way to review for the exam.

And then return to section 8.3, concerning conservative vector fields.

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Line integrals

Recall that we can apply line integrals to work problems:

If $F$ is a force field in the plane, and a path $c$ describes the motion of a particle in the plane, then

$$\int_c F \cdot ds$$

equals the contribution of work from the force field to the motion of the particle.

If $c$ is a simple closed curve, then Green’s Theorem gives a way to calculate that line integral; it’s equal to the double integral of the scalar curl of the field over the region that the path encloses.

In some cases, the double integral is much easier to calculate than the line integral.

In fact, sometimes we can calculate the line integral with Green’s Theorem without knowing exactly what the path is.

Example 1. Let $F = (\frac{x^2}{x^2+y^2}, \frac{x}{x^2+y^2})$, and let $c$ be ANY positively oriented simple closed curve that encloses the origin. Find

$$\int_c F \cdot ds.$$ 

For vector fields $F = (P,Q)$ that are defined everywhere in the plane, our second result from Wednesday boils down to this:

Check whether $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$.

If so,

- Line integrals on simple closed curves are 0.
- Line integrals between two given points are path-independent.
- There is a potential function $f$ such that $\nabla f = F$.

Now, how do we find that potential function?

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Now consider two paths (that are not closed) that go from $(1,0)$ to $(-1,0)$:

- $c_1(t) = (-t, 0), -1 \leq t \leq 1$
- $c_2(t) = (\cos t, \sin t), 0 \leq t \leq \pi$

Example 2a. Let $F = (x, x)$. Find $\int_c F \cdot ds$ along $c_1$ and along $c_2$.

Example 2b. Let $F = (x^2, 2xy)$, and let $c_1(t) = (-t, 2), -1 \leq t \leq 1$

$\quad \quad v_2(t) = (\cos t, 2 + \sin t), 0 \leq t \leq \pi$

Find $\int_c F \cdot ds$ along $c_1$ and along $c_2$. 

Example 3. Is \( \mathbf{F} = (3 + 2xy, x^2 - 3y^2) \) conservative? If so, find a potential function for \( \mathbf{F} \).

For vector fields in \( \mathbb{R}^3 \):

Check whether \( \text{curl } \mathbf{F} = 0 \).

Example 4. (8.3 #9) Find \( \int_c \mathbf{F} \cdot d\mathbf{s} \), where \( \mathbf{F} = (e^x \sin y, e^x \cos y, z^2) \) and \( \mathbf{c}(t) = (\sqrt{t}, t^3, e^{\sqrt{t}}) \) for \( 0 \leq t \leq 1 \).