Exam 2, #6. Let $F = (x^2, xy)$. Let $C^+$ be the positively oriented perimeter of the triangle that has vertices $(0, 0)$, $(3, 0)$, and $(0, 2)$. Find
\[ \int_{C^+} F \cdot ds. \]

Change of variables for double integrals
As we previewed on Wednesday, our goal here is to transform the region of integration so that the resulting double integral is easier to calculate.

(Think: in Calc I, we changed $dx$ to $du$. Here, we change $dx \, dy$ to $du \, dv$.)

And we saw that a factor is introduced in the integrand – the (absolute value of the) determinant of the derivative matrix of the transformation.

Let's state that formula more precisely now.

Change of variables for triple integrals
Suppose that $D^*$ and $D$ are elementary regions in $\mathbb{R}^3$, and that $T : D^* \to D$ is a one-to-one (except possibly on 2D subsets), onto, $C^1$ function. Then
\[
\iiint_{D^*} f(x(u,v,w), y(u,v,w), z(u,v,w)) |J| \, du \, dv \, dw = \iiint_{D} f(x, y, z) \, dx \, dy \, dz,
\]
where $J = |DT|$. 

Note: $J$ is the Jacobian, and the book denotes it as $\frac{\partial(x, y)}{\partial(u, v)}$.

Example 1. Let's test it out: Find the area of the parallelogram with vertices $(2, 1)$, $(4, 2)$, $(3, 5)$, and $(5, 6)$.
We discussed spherical coordinates Wednesday. Now,

**Cylindrical coordinates**

A point in \( \mathbb{R}^3 \) can be represented by the coordinates \((r, \theta, z)\), where \( r \geq 0 \) and \( 0 \leq \theta < 2\pi \). These coordinates are related to the rectangular coordinates of the point by

\[
x = r \cos \theta,
\]

\[
y = r \sin \theta,
\]

and

\[
z = z.
\]

**Example 2.** Find the volume of a solid tube of length 5, outer radius 4, and inner radius 3.

**Example 3.** Where does the map \( T(u, v) = (\cos u + \sin v, \cos u - \sin v) \) send the square \([0, \pi/2] \times [0, \pi/2]\)? Find the area of the square and its image.

**Example 4.** Find the integral of the function \( e^{-(x^2+y^2)} \) over the circle of radius \( R \).

**Why is the change of variables formula true?**

In other words, why is \(|J|\) in there?

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Find the area of the square and its image.

**Example 4.** Find the integral of the function \( e^{-(x^2+y^2)} \) over the circle of radius \( R \).