Today:

Review surface integrals of real-valued functions

Surface integrals of vector fields

Looking ahead to Stokes’ Theorem

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Integral of a real-valued function over a surface

**Definition:** The integral of a real-valued function $f(x,y,z)$ over a surface $S$ is defined to be

$$\int_S f \, dS = \iint_D f(\Phi(u,v)) \|T_u \times T_v\| \, du \, dv.$$  

**Example 1.** Find

$$\int_S e^z \, dS,$$

where $S$ is the part of the plane $x + y + z = 1$ that lies in the first octant (that is, where $x, y, z \geq 0$).

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Integral of a vector field over a surface

**Definition:** Suppose that a surface $S$ is parametrized by $\Phi : D \rightarrow \mathbb{R}^3$. The integral of a vector field $F$ in $\mathbb{R}^3$ over a surface $S$ is defined to be

$$\int_F \cdot dS = \int_D F(\Phi(u,v)) \cdot (T_u \times T_v) \, du \, dv.$$  

Note the difference in notation from surface integrals of real-valued functions.

**Example 2.** Given

$$F = \left(\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2}, 0\right),$$

find

$$\int_F \cdot dS,$$

where $\Phi$ is the standard parametrization for the unit sphere.

Physical interpretation?
Example 3. Let \( T(x, y, z) = x^2 + y^2 + z^2 \). Find the integral of \( \mathbf{F} = -\nabla T \) over the same sphere as in example 1.

(Compare answer to that of Example 4 on page 492 of the text.)