Remark: Please do at least five of the following twelve problems. Note that problems will be added as the course material progresses so make sure periodically re-check this file.

1) Exercises mentioned in Lecture 1:
   (a) Consider the representation $\rho: S_4 \to GL_2(\mathbb{C})$ given by

   $\rho((12)) = \rho((34)) = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}$,

   $\rho((13)) = \rho((24)) = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$,

   $\rho((14)) = \rho((23)) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

   Extending by multiplication (i.e. linearly), what do (123), (1324), (12)(34), and (132) map to under $\rho$?

   (b) Letting $g = (1234)$, $h = (132)$, verify that

   $\rho(g)^4 = \rho(h)^3 = \rho(ghh) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

   (b') Alternatively: verify the braid relations for $S_4$, i.e.

   $\rho((12))^2 = \rho((23))^2 = \rho((34))^2 = \rho((12)(34))^2 = \rho((12)(23))^3 = \rho((23)(34))^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
(c) Verify that the symmetric function (pattern repeats with an infinite number of variables)
\[x_1^2 x_2^2 + x_1^2 x_3^2 + x_2^2 x_3^2 + x_1^2 x_2 x_3 + x_1 x_2 x_3^2 + 2x_1 x_2 x_3 x_4 + \ldots\]
decomposes as
\[\frac{p_4^4}{12} - \frac{p_3 p_4}{3} + \frac{p_2 p_2}{4} = \frac{1}{24} \left( 1 \cdot 2 \cdot p_4^4 + 6 \cdot 0 \cdot p_2 p_1 + 8 \cdot (-1) \cdot p_3 p_1 + 6 \cdot 0 \cdot p_4 + 3 \cdot 2 \cdot p_2 p_2 \right).\]

2) (a) (Exercise 1 in Section 1.13 of Sagan) An \textit{inversion} in a permutation \(\pi = x_1, x_2, \ldots, x_n \in S_n\) (in one-line notation) is a pair \(x_i, x_j\) such that \(i < j\) and \(x_i > x_j\).

Show that if \(\pi\) can be written as a product of \(k\) transpositions, then \(k \equiv \text{inv}(\pi) \pmod{2}\). Note that this shows that the sign of \(\pi\), defined as \(\text{sgn}(\pi) = (-1)^k\), is well-defined.

(b) Exercise 7.3 in EC 2.

3) Exercise 110 in Chapter 3 of EC 1 (2nd edition). (Do at least part (a).)

4) Exercise 113 in Chapter 3 of EC 1 (2nd edition).

5) Exercise 114 in Chapter 3 of EC 1 (2nd edition). (Do at least parts (a) and (b).)

6) Exercise 115 in Chapter 3 of EC 1 (2nd edition). (Note that parts (b) and (c) of this problem are difficult.)

The next two problems are from Staley’s Notes on Hyperplane Arrangements, available at http://math.mit.edu/~rstan/arrangements/arr.html:

7) (a) Lecture 1, Exercise 2 (page 13).

(b) Lecture 1, Exercise 3 (page 13).

(c) Lecture 2, Exercise 3 (page 31).
8) (a) Lecture 2, Exercise 4 (page 31).
   (b) Lecture 2, Exercise 7 (page 31).
   (c) Lecture 2, Exercise 8 (page 31).

9) Exercise 5.1 in EC 2 (1st edition).

10) Exercise 5.3 in EC 2 (1st edition).

11) Exercise 5.6 in EC 2 (1st edition).

12) Exercise 5.10 in EC 2 (1st edition). (Do at least parts (a), (b), and (c).)

   No more problems to be added at this point.