I encourage collaboration on the homework, as long as each person understands the solutions, writes them up in their own words, and indicates on the homework page their collaborators.

Please do at least five of the following nine problems. (Front and Back)

1) Let $Q$ be the quiver $1 \xrightarrow{\alpha} 2$. (Here $\alpha$ is meant to be the name or value of this arrow.)
   (a) Show that the only indecomposable representations are
       
       $S_1 = K \rightarrow 0, \quad S_2 = 0 \rightarrow K, \quad$ and $\quad P_1 = K \rightarrow 1 K$.
   
   (b) Suppose that $M$ is a representation of $Q$ with vector spaces $M(1) = K^n$ and $M(2) = K^m$ and linear
       map $M(\alpha) = A : K^n \rightarrow K^m$.
       Show that $M \cong S_1^{d_1} \oplus S_2^{d_2} \oplus P_1^{r}$ where $d_1$ is the dimension of the kernel of $A$, $d_2$ is the dimension of the
       cokernel of $A$, and $r$ is the rank of $A$.
   
   (c) Show that the representation $P(1)$ is not simple (i.e. irreducible).

2) Problem 1.3 of Schiffler

3) Let $Q$ be the quiver $1 \xleftarrow{\alpha} 2 \xrightarrow{\beta} 1$ (i.e. with two arrows: $\alpha$ from 1 to 2 and $\beta$ from 2 to 1). Show that the path
    algebra $KQ$ is isomorphic to the quotient of the free associative algebra $K(\alpha, \beta)/ (\alpha^2, \beta^2)$.

4) Problem 1.6 of Schiffler

5) Problem 2.6 of Schiffler

6) Let $Q_n$ be the unidirectional quiver of type $A_n$ which consists of $n$ vertices linearly ordered and labeled
   as $\{1, 2, \ldots, n\}$, and arrows $a_i$ between vertex $i$ and $(i + 1)$. Let $k$ be a field and $kQ_n$ be the associated
   path algebra. Let $B_n(k)$ denote the $k$-algebra of lower-triangular $n$-by-$n$ matrices over field $k$.
   a) Show that $kQ_n$ and $B_n(k)$ are isomorphic as $k$-algebras.

   **Hint:** It suffices to exhibit a map between basis elements and then show that the same relations are
   satisfied in both $k$-algebras.

   b) Describe the projective and injective indecomposable representations of $Q_n$. 
7) Problem 3.1 of Schiffler for the two quivers of type $A_6$, i.e. parts (1) and (2).

8) Problem 2.11 of Schiffler

9) Problem 2.12 of Schiffler