Call the printed area $A$ and the total paper area $P$ (which we want to minimize).

$A = hL = 384 \text{cm}^2$

$P = (h + 6\text{cm} + 6\text{cm})(L + 4\text{cm} + 4\text{cm})$

$= (h + 12\text{cm})(L + 8\text{cm})$

$= hL + 12\text{cm}L + 8\text{cm}h + 96\text{cm}^2$

$= 384\text{cm}^2 + 12\text{cm}L + 8\text{cm}h + 96\text{cm}^2$

So $P = 348\text{cm}^2 + 12\text{cm}L + 8\text{cm}h + 384(\frac{3}{L}) + 96\text{cm}^2$

$\frac{dP}{dL} = P' = 0 + 12\text{cm} + 8\text{cm} \cdot 384(\frac{-1}{L^2})$

$= 12 - \frac{8 \cdot 384}{L^2}$

Min & Max occur when $P' = 0 \Rightarrow 12 = \frac{8 \cdot 384}{L^2}$

So 16 cm is a min.

$12 = \frac{8 \cdot 384}{L^2}$

$L^2 = \frac{8 \cdot 384}{12}$

$L^2 = 2 \cdot \frac{\sqrt{128}}{3}$

$L = 2 \cdot \frac{\sqrt{\sqrt{128}}}{3}$

Test points: Need an $L > 0$, $16$ and an $L < 16$.

To plug into $P'$: $12 - \frac{8 \cdot 384}{L^2}$

Choose an $L$ we're okay with dividing:

$L = 1 \Rightarrow P'(1) = 12 - 8 \cdot 384 < 0$ Then choose a power of 2; because $384 = 3 \cdot 128 = 3 \cdot 2^7$. Choose $L = 32$:

$P'(32) = 12 - \frac{8 \cdot 32}{32} = 12 - \frac{8 \cdot 32}{32} = 12 - \frac{3 \cdot 2^7}{32} = 12 - \frac{3 \cdot 32}{32} = 12 - 3 > 0$
31 (cont'd)

If \( L = 16 \text{cm} \), then \( h = \frac{3.84 \text{cm}^2}{16 \text{cm}} = \frac{3.128 \text{cm}}{16} = 0.198 \text{cm} \)

So the total dimensions are:

- \( h + 12 \text{cm} = 36 \text{cm} \)
- \( L + 8 \text{cm} = 24 \text{cm} \)

4.8

11. \( 5^{\sqrt{20}} \) is the zero of some function \( f(x) \).

So \( f(5^{\sqrt{20}}) = 0 \).

\[ [5^{\sqrt{20}}]^5 = 20 \]

\[ [5^{\sqrt{20}}]^5 - 20 = 0 \]

So \( f(x) = x^5 - 20 \),

we'll need \( f'(x) = 5x^4 \).

We guess \( x_1 = 2 \) (because \( 2^5 = 32 \), but it's closer
to 20 than either \( 1^5 \) or \( 3^5 \)).

\[ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{2^5 - 20}{5 \cdot 2^4} = 2 - \frac{12}{80} = \frac{37}{20} = 1.85 \]

\[ x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.85 - \frac{1.85^5 - 20}{5 \cdot 1.85^4} = 1.821486 \]

\[ x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.820564203 \]

\[ x_5 = 1.820564203 \]

\[ x_6 = 1.820564203 \]

Because 8 digits stayed the same
between 2 passes through Newton's Method.
31) \[ f(x) = \sqrt{x}(6 + 5x) = 6\sqrt{x} + 5x^{3/2} \]

**Rule:** If \( f(x) = x^n \)
then \( g(x) = \frac{x^{n+1}}{n+1} \)

\[ f(x) = 6 \cdot \frac{x^{(1/2+1)}}{1/2+1} + 5 \cdot \frac{x^{(3/2+1)}}{(3/2+1)} + c = 6 \cdot \frac{x^{3/2}}{3/2} + 5 \cdot \frac{x^{5/2}}{5/2} + c \]

\[ = 6 \cdot \frac{2}{3} x^{3/2} + 5 \cdot \frac{2}{5} x^{5/2} + c = 4 x^{3/2} + 2 x^{5/2} + c \]

**Check:** \( f'(x) = \frac{3}{2} \cdot 4x^{1/2} + \frac{5}{2} \cdot 2x^{3/2} + 0 = 6x^{1/2} + 5x^{3/2} \checkmark \)

37) \( f^0(x) = 24x^2 + 2x + 10, \quad f(1) = 5, \quad f'(1) = -3 \)

\[ f'(x) = 24 \cdot \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} + 10x^1 + c = 8x^3 + x^2 + 10x + c \]

\[ f'(1) = 8 \cdot 1^3 + 1^2 + 10 + c = 19 + c = -3 \]

\[ \Rightarrow c = -22 \]

\[ f'(x) = 8x^3 + x^2 + 10x - 22 \]

\[ f(x) = 8 \cdot \frac{x^4}{4} + \frac{x^3}{3} + 10 \cdot \frac{x^2}{2} - 22 \cdot \frac{x^1}{1} + d \]

\[ = 2x^4 + \frac{x^3}{3} + 5x^2 - 22x + d \]

\[ f(1) = 2 \cdot 1^4 + \frac{1^3}{3} + 5 \cdot 1^2 - 22 \cdot 1 + 5d = 2 + \frac{1}{3} + 5 - 22 + d \]

\[ = \frac{1}{3} - 15 + d = 5 \]

\[ d = 20 - \frac{1}{3} = \frac{59}{3} \]

\[ f(x) = 2x^4 + \frac{x^3}{3} + 5x^2 - 22x + \frac{59}{3} \]