1. Suppose you need to build a closed box with a square base, and you have 2400 cm\(^2\) of material to use. What is the largest possible volume you can make this box, and what are the corresponding dimensions of the box?

\[
\begin{align*}
S &= 2x^2 + 4xh = 2400 \text{ cm}^2 \\
h &= \frac{2400 - 2x^2}{4x} = \frac{600}{x} - \frac{x}{2}
\end{align*}
\]

\[
V = x^2 h = x^2 \left(\frac{600}{x} - \frac{x}{2}\right) = 600x - \frac{x^3}{2}
\]

\[
V' = 600 - \frac{3}{2}x^2 \quad V' = 0 \Rightarrow 600 = \frac{3}{2}x^2 \Rightarrow x^2 = \frac{2}{3} \cdot 600
\]

\[
x^2 = \frac{2}{3} \cdot 600
\]

\[
x = \pm 20
\]

2. Use Newton's method with initial approximation \(x_1 = -1\) to find the third approximation \(x_3\) of the root of \(x^5 + 2 = 0\).

\[
f(x) = x^5 + 1 \quad f'(x) = 5x^4
\]

\[
x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -1 - \frac{(-1)^5 + 2}{5(-1)^4} = -1 - \frac{-1 + 2}{5} = -1 - \frac{1}{5} = -\frac{6}{5}
\]

\[
x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \left(-\frac{6}{5}\right) - \frac{\left(-\frac{6}{5}\right)^5 + 2}{5\left(-\frac{6}{5}\right)^4}
\]

3. Find \(f\) if \(f'(x) = 8x^3 + 12x + 3\) and \(f(1) = 6\).

\[
f(x) = 8 \cdot \frac{x^4}{4} + 12 \cdot \frac{x^2}{2} + 3 \cdot \frac{x}{1} + C
\]

\[
= 2x^4 + 6x^2 + 3x + C
\]

\[
f(1) = 2 \cdot 1^4 + 6 \cdot 1^2 + 3 \cdot 1 + C = 2 + 6 + 3 + C
\]

\[
= 11 + C = 6
\]

\[
C = -5
\]

\[
f(x) = 2x^4 + 6x^2 + 3x - 5
\]