For all questions, show your work. State which tests you are using. Put answers in reasonably simple formats.

1. Find an equation for the line through the points (2, 1, 3) and (4, 0, 5).

\[ L_1 = t(2, 1, 3) + (1-t)(4, 0, 5) = (4, 0, 5) + (\langle 2, 1, 3-5 \rangle t \]

\[ = \langle 4-2t, 5-2t \rangle \quad \text{on} \quad x = 4-2t, \quad y = t, \quad z = 5-2t \]

2. Find an equation for the plane containing the points (2, 1, 3), (4, 0, 5) and (1, 1, 1).

\[ \vec{a} = \langle 2-4, 1-1, 3-5 \rangle = \langle 2, 0, -2 \rangle \]

\[ \vec{b} = \langle 2-1, 1-1, 3-1 \rangle = \langle 1, 0, 2 \rangle \]

\[ \mathbf{n} = \det \begin{vmatrix} \hat{i} & \hat{j} \\ 2 & 1 \\ 1 & 0 \end{vmatrix} = 2\hat{j} - 2\hat{k} + 0\hat{k} - (-2\hat{i} + 0\hat{j} + 1\hat{k}) = 2\hat{i} + 2\hat{j} - \hat{k} = \langle 2, 2, -1 \rangle \]

\[ 2(x-1) + 2(y-1) - (z-1) 
\]

\[ = 2x - 2 + 4y - 2 - z + 1 = 0 \]

\[ \Rightarrow 2x + 4y - z = 3 \]

\[ 2x + 2y - z = \square \]

\[ \text{unnecessary, but a check:} \quad \langle 4, 0, 5 \rangle: \quad 2(4) + 2(0) - 5 = 8 - 5 = 3 \]

\[ \langle 2, 1, 3 \rangle: \quad 2(2) + 2(1) - 3 = 6 - 3 = 3 \]
3. State whether the following series converges or diverges. \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}} \)

Compare with \( \sum_{n=1}^{\infty} \frac{1}{n} \), which diverges by the p-series test.

\[
\lim_{n \to \infty} \left( \frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{n}} \right) = \lim_{n \to \infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n \to \infty} \frac{n \cdot \frac{1}{n}}{\sqrt{n^2+1} \cdot \frac{1}{n}} = \lim_{n \to \infty} \frac{1}{\sqrt{1 + \frac{1}{n^2}}} = \frac{1}{1} = 1
\]

So we can use the comparison. One series diverges, so they both diverge.

4. Solve the following differential equation, putting your answer in as simple of terms as possible:

\[
(1 + \cos(y)) \, dy = (x^2 + 1) \, dx
\]

\[
\int (1 + \cos(y)) \, dy = y + \sin y = \int (x^2 + 1) \, dx = \frac{x^3}{3} + x + C
\]

\( y(1) = 0 \) \quad \Rightarrow \quad \frac{x^3}{3} + x + C = 0 \quad \Rightarrow \quad x = -\frac{4}{3}

So \( y + \sin y = \frac{x^3}{3} + x - \frac{4}{3} \).

5. Evaluate the integral \( \int \cot(x) \, dx \) using methods of integration.

\[
\int \cot(x) \, dx = \int \frac{\cos x}{\sin x} \, dx = \int \frac{\frac{dw}{u}}{u} = \ln |u| + C
\]

\( u = \sin x \)

\( \frac{dw}{u} = \cos x \, dx \)