1. State the two parts of the hypothesis and conclusion for the mean value theorem for the function \( f(x) = (x - 8)^{2/3} \) on the interval \( 0 \leq x \leq 35 \). Which of these statements are true?

2. Let \( f(x) = x(2 + x) \) on the interval \( 0 \leq x \leq 5 \), that is, \( a = 0 \) and \( b = 5 \).
   a) Find \( (a + \frac{2k-1}{2n}(b - a)) \).
   b) Find \( f(a + \frac{2k-1}{2n}(b - a)) \).
   c) Find \( \sum_{k=1}^{n} f(a + \frac{2k-1}{2n}(b - a)) \) and simplify.
   d) Find \( \lim_{n \to \infty} \frac{b-a}{n} \sum_{k=1}^{n} f(a + \frac{2k-1}{2n}(b - a)) \).

3. Set up the integral which when evaluated is equal to the area of the region bounded by the curves \( y = x^2 - 9 \) and \( y = -2x + 6 \) and between the lines \( x = -3 \) and \( x = 5 \).
1. Set up the integral which when evaluated will equal the following volume. Find the volume generated when the region bounded by the curves \( y = x^2 \) and \( y = 4x + 12 \) is revolved about the line \( y = -3 \).

2. Set up the integral which when evaluated will equal the following volume. Find the volume generated when the region bounded by the curve \( y = -x^2 + 9x - 14 = -(x-2)(x-7) \) and the \( x \) axis is revolved about the line \( x = 10 \).

3. A bucket that weighs 150 lbs when filled with water is lifted by a mechanical winch from the bottom of a well that is 120 feet deep. The chain that is being used to lift the bucket weighs 1/2 pounds per foot. There is a hole in the bucket. As the bucket is raised water leaks from the bucket at a constant rate of 3/4 lbs/ft, that is, for every foot the bucket moves up the weight of water in the bucket decreases by 3/4 lb. When it reaches the top of the well the bucket weighs 60 lbs. Note that in this situation, the weight of the bucket and water is a linear function of how far the bucket has moved. Find the work required to roll up the chain on the mechanical winch from the point where the bucket is 20 feet from the bottom to the point where it is 80 feet from the bottom.
5845 A Third Review

1. Let \( f(x) = \begin{cases} 
3x^2 - 4x - 13 & x < 3 \\
2x - 4 & x \geq 3 
\end{cases} \) Find \( F(x) \) such that \( F'(x) = f(x) \) for all \( x \). Note that \( f(x) \) is continuous for all \( x \).

2. Evaluate the definite integral \( \int_0^7 |x^2 - 10x + 21| \, dx \).

3. Find the antiderivative \( \int \frac{\sin(5x) \cos(5x)}{\sqrt{4 + \sin^2(5x)}} \, dx \) using appropriate \( u \)-substitution.

4. Given that \( \sum_{k=1}^{n} (2k - 1) = n^2 \) and \( \sum_{k=1}^{n} (2k - 1)^2 = \frac{n}{3} (4n^2 - 1) \), find \( \int_1^6 x^2 \, dx \) using the definition of the definite integral. First, you need to find \( \sum_{k=1}^{n} f\left( a + \frac{2k - 1}{2n}(b - a) \right) \).
1. Find the linearization \( L(x) \) of the function \( f(x) = \sqrt{4x + 1} \) at the point where \( x = 6 \). Evaluate \( L(x) \) at \( x = 6.05 \) and so obtain an estimate of the value of \( \sqrt{25.2} \).

2. A water tank is in the shape of a cone with the tip at the top. The height of the tank is 10 feet and the diameter of the base is 12 feet. How much work is required to pump all the water to a level 5 feet above the top of the tank?

3. A particle is moving back and forth along a line. Its velocity is given by \( v(t) = 8t - t^2 \) ft/min. Find the total distance it travels during the time interval \( 0 \leq t \leq 12 \).

4. Find the total area of the region in the \( xy \) plane bounded by the curve \( y = \frac{1 - x^2}{2(x^2 + 1)} \), the \( x \) axis, the \( y \) axis, and the line \( x = \sqrt{3} \). Note that there are two parts to this region. Is 0.3896 correct?