1. A large water tank is in the shape of an inverted cone, the way you hold a cone to eat ice cream. The cone has been truncated. The bottom half is not there. The whole cone would be 10 feet high but the bottom half of the cone is gone. The remaining tank is just 5 feet deep. The radius of the top of the tank is 4 feet and the radius of the bottom is 2 feet. Water is leaking out of the bottom of the tank at the rate of $3\pi \, \text{ft}^3$ per minute. How fast is the height (depth) of water in the cone changing when the water in the tank is 2.5 ft deep?

$$V = \text{Volume of water in tank}$$

$$V = \frac{1}{3} \pi (x^2) (5+y) - \frac{1}{3} \pi (2^2) (5)$$

$$V = \frac{4\pi}{75} (5+y)^3 - \frac{20\pi}{3}$$

$$\frac{dV}{dt} = \frac{4\pi}{75} (5+y)^2 \frac{dy}{dt}$$

$$-3\pi = \frac{4\pi}{75} (7.5)^2 \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{-1}{3} \frac{\text{ft}}{\text{min}}$$

2. The function $f(x)$ is a continuous function defined only for $-5 < x < 10$ with $f'(-5) > 0$, $f'(0) = 0$, $f'(5) = \text{DNE}$ and $f'(10) > 0$. The first derivative $f'(x)$ satisfies $f'(x) > 0$ for $-5 < x < 0$ and $5 < x < 10$. Also $f'(x) < 0$ for $0 < x < 5$. Find all the local maximum and minimum points for $f(x)$.

$$f'(x) > 0 \quad f'(x) > 0 \quad f'(0) = 0 \quad f'(x) < 0 \quad f'(x) = \text{DNE} \quad f'(x) > 0 \quad f'(10) > 0$$

$f(x)$ has a local minimum at $x = -5$ by left-end point theorem.

$f(x)$ has a local maximum at $x = 0$ by first derivative test.

$f(x)$ has a local minimum at $x = 5$ by first derivative test.

$f(x)$ has a local maximum at $x = 10$ by the right end point theorem.

3. Use the direct method found in Example 5 of 6355 to show that $f(x) = x^3 - 12x^2 + 45x$ has a local maximum value at $x = 3$.

$$f'(x) - f'(3) = x^3 - 12x^2 + 45x - 54 = (x-3)^2 (x-6)$$

If $x < 6$, then $f'(x) - f'(3) < 0$

If $x > 6$, then $f'(x) - f'(3) > 0$

For $x$ near 3, $f'(x) < f'(3)$

$f(x)$ has a local maximum at $x = 3$.