5806 Another Newton's Method

1. Given that \( \sum_{k=1}^{n} (2k - 1) = n^2 \) and \( \sum_{k=1}^{n} (2k - 1)^2 = \frac{n}{3} (4n^2 - 1) \), find

a) \[ \lim_{n \to \infty} \frac{6}{n} \sum_{k=1}^{n} \left[ \frac{18}{n^2} (2k-1)^2 + 3 \right] = \frac{6}{n} \left[ \frac{18}{n^2} \frac{n}{3} (4n^2 - 1) + 3n \right] \]

\[ \lim_{n \to \infty} \left( \frac{162}{n^2} - \frac{36}{n} \right) = 162 - \frac{36}{n} = 162 \]

b) \[ \lim_{n \to \infty} \frac{6}{n} \sum_{k=1}^{n} \left\{ \left[ \frac{3(2k-1)}{n} \right]^2 + 4 \left[ \frac{3(2k-1)}{n} \right] \right\} = \lim_{n \to \infty} \left( 144 - \frac{9}{n^2} \right) = 144 \]

2. Solve \( \sqrt{2x^3 + 5x^2 - 8x} = \sqrt{x^3 + 10x + 24} \) by first squaring both sides and then using Newton's Method. Use \( x_1 = 3 \) as your first guess in Newton's Method. The number \( x = -7.0666 \) is a solution of the cubic. Is it also a solution of the given equation?

**Squaring both sides gives** \( x^3 + 5x^2 - 18x - 24 = 0 \).

\( x_1 = 3 \), \( x_2 = 3.153846154 \), \( x_3 = 3.146123417 \)

The roots, i.e. 3.146123413442

Using synthetic division, the other two roots are solutions of

\( x^2 + 8.146103442 \cdot x + 7.6284844076 = 0 \)

The roots are

\(-7.066589134 \) and \(-1.079514308 \)

Substituting \( x = -7.0666 \) into the original equation gives \( \sqrt{399.5479} = \sqrt{399.5479} \). Since we are dealing only with real numbers, we assume that numbers inside the square root sign must be positive,