5811 Definition of Definite Integral

1. Let \( f(x) = x^2, \ a = 1, \ b = 5, \) find \( \sum_{k=1}^{n} f(a + \frac{2k-1}{2n}(b-a)) \) when \( n = 4. \)

\[
\begin{align*}
X_k &= a + \frac{2k-1}{2n}(b-a) = k + \frac{1}{2}, \quad k = 1, 2, 3, 4 \\
F\left(\frac{3}{2}\right) &= \frac{9}{4} \quad F\left(\frac{5}{2}\right) = \frac{25}{4} \quad F\left(\frac{7}{2}\right) = \frac{49}{4} \quad F\left(\frac{9}{2}\right) = \frac{81}{4} \\
\sum_{k=1}^{n} F\left(a + \frac{2k-1}{2n}(b-a)\right) &= \frac{9}{4} + \frac{25}{4} + \frac{49}{4} + \frac{81}{4} = 41
\end{align*}
\]

2. If \( f(x) = x^2, \ a = 0, \) and \( b = 6, \) find \( F(a + \frac{2k-1}{2n}(b-a)) \) for any \( n. \) If \( \sum_{k=1}^{n} (2k-1)^2 = \frac{n}{3}(4n^2 - 1) \) find \( \frac{b-a}{n} \sum_{k=1}^{n} f(a + \frac{2k-1}{2n}(b-a)). \)

\[
\begin{align*}
0 + \frac{2k-1}{2n}(6) &= \frac{3(2k-1)}{n} \\
&= \left( a + \frac{2k-1}{2n}(b-a) \right) \\
\frac{b-a}{n} \sum_{k=1}^{n} F\left(a + \frac{2k-1}{2n}(b-a)\right) &= \frac{6}{n} \sum_{k=1}^{n} \frac{9}{n^2} (2k-1)^2 \\
&= \frac{54}{n^3} \left[ \frac{n}{3} (4n^2-1) \right] = 72 - \frac{18}{n^2}
\end{align*}
\]

3. Find the area of the region bounded by the curve which is the graph of \( f(x) = 10x - x^2, \) the \( x \) axis, and both the lines \( x = 2 \) and \( x = 8. \)

\[
\int_{2}^{8} (10x - x^2) \, dx = 132
\]

4. A particle is moving along a straight line with velocity given by \( v = 6t^2 + 4t + 5 \) feet per second where \( t \) is time. Find the distance the particle moves during the time interval \( 2 \leq t \leq 5. \) Note that \( v \geq 0. \)

\[
\int_{2}^{5} (6t^2 + 4t + 5) \, dt = 291
\]