5846 A Fourth Review

1. Find the linearization \( L(x) \) of the function \( f(x) = \sqrt{4x + 1} \) at the point where \( x = 6 \). Evaluate \( L(x) \) at \( x = 6.05 \) and so obtain an estimate of the value of \( \sqrt{25.2} \).

\[
L(x) = 5 + \frac{2}{5} (x-6)
\]

\[
L(6.05) = 5 + \frac{2}{5}(0.05) = 5.02
\]

2. A water tank is in the shape of a cone with the tip at the top. The height of the tank is 10 feet and the diameter of the base is 12 feet. How much work is required to pump all the water to a level 5 feet above the top of the tank?

Radius = \( x = 6 - \frac{3}{5} y \)

\[ \Delta V = \pi \left[ 6 - \frac{3}{5} y \right]^2 \Delta y \]

\[ \text{Work} = \frac{\rho \pi}{2 \cdot 5} \int_0^5 (30-3y)^2(15-y) \, dy \]

\[ = 1500 \pi \rho = 294,524 \text{ ft-lbs} \]

3. A particle is moving back and forth along a line. Its velocity is given by \( v(t) = 8t - t^2 \) ft/min. Find the total distance it travels during the time interval \( 0 \leq t \leq 12 \).

\[
\int_0^{12} |8t-t^2| \, dt = \int_0^8 (8t-t^2) \, dt - \int_8^{12} (8t-t^2) \, dt
\]

\[ = \frac{256}{3} + \frac{256}{3} = \frac{512}{3} \text{ ft} \]

4. Find the total area of the region in the \( xy \) plane bounded by the curve \( y = \frac{1 - x^2}{2(x^2+1)} \), the \( x \) axis, the \( y \) axis, and the line \( x = \sqrt{3} \). Note that there are two parts to this region. Is 0.3896 correct?

\[
\int_0^1 \frac{1-x^2}{2(x^2+1)} \, dx = -\frac{x}{2} + \arctan x + C
\]

\[
\int_0^1 \frac{1-x^2}{2(x^2+1)} \, dx = \int_{\sqrt{3}}^1 \frac{1-x^2}{2(x^2+1)} \, dx
\]

\[ = \frac{\pi}{4} - \frac{1}{2} + \frac{\sqrt{3}}{2} - \frac{\pi}{3} + \frac{\pi}{4} - \frac{1}{2} = 0.3896 \]