5956 Review of Velocity

1. The position of a particle at time \( t \) is given by \( s(t) = t^3 - 8t^2 + 16t \) for \( t \geq 0 \).

(a) Find the velocity at time \( t \). Recall that velocity is the rate of change of position with respect to time.

\[
\sqrt{t} = s'(t) = \frac{ds}{dt} = 3t^2 - 16t + 16.
\]

(b) What is the velocity when \( t = 3 \)?

\[
\sqrt{3} = 27 - 48 + 16 = 27 - 32 = -5.
\]

(c) When is the particle at rest?

\[
\sqrt{t} = 0 \iff 3t^2 - 16t + 16 = 0 \iff t = \frac{16 \pm \sqrt{16^2 - 3 \cdot 4 \cdot 16}}{6} = \frac{16 \pm \sqrt{16(16 - 12)}}{6} = \frac{16 \pm 4\sqrt{4}}{6} = \frac{16 \pm 8}{6} = \frac{4}{3} \text{ or } 4
\]

(d) When is the particle moving in a negative direction?

The velocity is a quadratic equation, opening upward. If a parabola hits \( 0 \) twice, it is positive on one side of the \( 0 \) and negative on the other. Also, \( \sqrt{3} < 0 \), so

2. Suppose \( y \) is defined as a function of \( x \) by the equation \( x^{3/2}y^2 - 2x^2y = 5x + 20 \). How many points on this curve have \( x \) coordinate 4? Find \( \frac{dy}{dx} \) at all points with \( x \) coordinate \( x = 4 \). \( \frac{\frac{3}{2}x^{1/2}y^2 + 2x^{3/2}y}{2x^2}y' - xy - 2x^2y' = 5 \).

\[
\frac{x = 4, y = 5}{\frac{3}{2} \cdot 4 \cdot 5 + 2 \cdot 4 \cdot 5 \cdot y' - 4 \cdot 5 - 2 \cdot 4^2 \cdot y' = 5} \implies \frac{y' = \frac{100}{-24}}{y' = \frac{5}{24}}
\]

3. A steel ball is heated to a temperature of 320\(^\circ\) and placed in a room where the temperature is 70\(^\circ\). After 5 minutes the temperature of the ball is 194. According to Newton’s law of cooling the temperature \( u(t) \) of the ball at any time \( t \) is given by

\[
u(t) = 70 + 250e^{-0.14t}.\]

What is the rate of change of temperature with respect to time when \( t = 5 \) minutes?

\[
u'(t) = 250e^{-0.14t} \cdot (-0.14) = -350e^{-0.14t} \quad \text{and} \quad u'(5) = -35e^{-0.14 \cdot 5} = -35e^{-0.7}.
\]