5994 Review Derivative Tests

1. Let \( f(x) = x^{5/3} - 20x^{2/3} \). Find the critical values of \( x \) for \( f(x) \). Use the first derivative test to determine which critical values give a local maximum and which a local minimum for \( f(x) \). Second, test values using the second derivative test.

\[
\frac{d}{dx} f(x) = \frac{5}{3} x^{-1/3} (x-8) \quad \frac{d}{dx} f(8) = 0 \quad f'(0) = \text{DNE}
\]

- \( f'(x) > 0 \) for \( x < 0 \) and \( f'(x) < 0 \) for \( x > 8 \)

**First derivative test** - \( f(x) \) has a local maximum value at \( x = 0 \) and a local minimum value at \( x = 8 \).

\[
\frac{d^2}{dx^2} f(x) = \frac{10}{9} x^{-4/3} + \frac{20}{9} x^{-4/3} \quad \frac{d^2}{dx^2} f(0) = \text{DNE} \quad \frac{d^2}{dx^2} f(8) = \frac{5}{9}
\]

**Second derivative test** fails at \( x = 0 \) and gives a local minimum at \( x = 8 \).

2. The function \( f(x) \) is continuous and has the following properties

(a) \( f'(-2) = 0, f'(3) = 0, f'(10) = \text{DNE}, f''(0) = 0, f''(3) = 0, f''(10) = \text{DNE} \)

(b) \( f'(x) > 0 \) for \(-2 < x < 3\), and for \(3 < x < 10\). Also \( f'(x) < 0 \) for \( x < -2 \) and for \( x > 10 \)

(c) \( f''(x) > 0 \) for \( x < 0 \), and for \( 3 < x < 10 \), and for \( x > 10 \). Also \( f''(x) < 0 \) for \( 0 < x < 3 \).

This function has 3 critical points. What are they? First, using the second derivative test (which is easier) where does the function \( f(x) \) have a local maximum or a local minimum.

The three critical points are \( x = -2, 3, \) and \( 10 \).

- \( f''(-2) > 0 \) local min at \( x = -2 \)
- \( f''(3) = 0 \) test fails at \( x = 0 \)
- \( f''(10) = \text{DNE} \) test fails at \( x = 10 \)

Second, do the whole problem over again. Use the first derivative test to tell if the function \( f(x) \) has a local maximum or a local minimum at the 3 critical points.

\[
\begin{array}{cccc}
 f'(x) < 0 & f'(x) > 0 & f'(x) > 0 & f'(x) < 0 \\
 -2 & 3 & 10 \\
\end{array}
\]

**The first derivative test says**

- Local minimum value at \( x = -2 \)
- Neither maximum or minimum value at \( x = 3 \)
- Local maximum at \( x = 10 \)