1. Find the limit, if it exists, or show that it does not exist.
\[
\lim_{(x,y) \to (0,0)} \frac{5y^4 + (\cos x)^2}{x^4 + y^4}
\]

2. \( \vec{F} \circ (x,y,z) = \langle x, y, 5 \rangle = x \hat{i} + y \hat{j} + 5 \hat{k} \).
   
   \( S \) is the boundary of the region enclosed by the cylinder \( x^2 + z^2 = 1 \) and the planes \( y = 0, \ x + y = 2 \).

3. Find the flux. \( \vec{F}(x,y,z) = x^2 \hat{i} + y^2 \hat{j} + z^2 \hat{k} \).
   
   \( S \) is the boundary of the solid half cylinder
   \( 0 \leq z \leq \sqrt{1 - y^2}, \quad 0 \leq x \leq 2 \).

4. Find a function \( f \) such that \( \vec{F} = \nabla f \) and use it to evaluate \( \int_C \vec{F} \cdot d\vec{s} \) along \( C \).
   
   \( \vec{F} = \sin y \hat{i} + (x \cos y + \cos z) \hat{j} - (y \sin z) \hat{k} \)
   
   \( C: \vec{r}(t) = (\sin t) \hat{i} + t \hat{j} + 2t \hat{k}, \quad 0 \leq t \leq \pi/2 \).

5. Using Lagrange Multipliers, find the dimensions of the box with largest volume if the total surface area is 64 cm\(^2\).
6. Evaluate the integral by making the appropriate change of variables:

\[ \iint_R \cos \left( \frac{y-x}{x+y} \right) dA \]

where \( R \) is the trapezoidal region with vertices \((1,0); (2,0); (0,2); (0,1)\).

7. If \( u = f(x,y) \), where \( x = e^s \cos t \), \( y = e^s \sin t \), show that

\[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2s} \left[ \frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2} \right] \]

8. Find the tangent plane to the surface given by

\[ z = 3x^2 + 3y^2 \]

at the point \((3,4,5\sqrt{3})\).

9. A region \( R \) in the \( xy \) plane is given. Find equations for a transformation \( T \) that maps a rectangular region \( S \) in the \( uv \) plane onto \( R \), where the sides of \( S \) are parallel to the \( u \) and \( v \) axes.

\( R \) is bounded by the hyperbolas \( y = \frac{1}{x}, \) \( y = \frac{4}{x} \), and the lines \( y = x, \) \( y = 4x \), in the first quadrant.
10. Prove the identity, assuming all appropriate partials exist & are continuous:

\[ \text{div} (\vec{E} \otimes \vec{G}) = \vec{G} \cdot \text{curl} (\vec{E}) - \vec{E} \cdot \text{curl} (\vec{G}) \]

11. Evaluate the surface integral \( \iiint _S \vec{F} \cdot d\vec{S} \), where

\[ \vec{F} = xy\hat{i} + yz\hat{j} + zx\hat{k} \]

\( S \) is the part of the paraboloid \( z = 4 - x^2 - y^2 \), that lies above the square \( 0 \leq x \leq 1, 0 \leq y \leq 1 \) with upward orientation.

12. \( f(x, y, z) = 3x - y - 3z, \quad x + y - z = 0, \quad x^2 + 2z^2 = 1 \).

Find the extreme values of \( f \), subject to both constraints.

13. \( z = x \ln(xy) + y^3, \quad y = \cos(x^2 + 1) \).

Find \( \frac{\partial z}{\partial x} \).

14. Let \( \vec{F} = -2xy\hat{i} + y^2\hat{k} \).

(a) Calculate \( \nabla \times \vec{F} \).

(b) Show that \( \iint _R \text{curl} \vec{F} \cdot n \, dS = 0 \) for any finite portion \( R \) of the unit sphere \( x^2 + y^2 + z^2 = 1 \).

(c) Show that \( \oint _C \vec{F} \cdot d\vec{r} = 0 \) for any closed curve \( C \) on the unit sphere \( x^2 + y^2 + z^2 = 1 \).
15. Find the area of the surface created when we take the boundary of the intersection of \( y^2 + \theta z^2 = 1 \) and \( x^2 + z^2 = 1 \).

![Diagram]

16. Let \( S \) be a disk of radius 6 centered around the \( z \)-axis in plane \( z = -4 \), oriented with an upward pointing normal. Let a magnetic field be given by \( \vec{F}(x,y,z) = (0,0,x^2+y^2) \).

What is the total magnetic flux through the disk?

16. Find the volume of the largest box in the first octant with the three faces in the coordinate planes and one vertex in the plane \( x + 2y + 3z = 6 \).

17. If \( z = f(x,y) \) where \( x = r\cos\theta \), \( y = r\sin\theta \), find

(a) \( \frac{\partial z}{\partial r} \)
(b) \( \frac{\partial z}{\partial \theta} \)
(c) \( \frac{\partial^2 z}{\partial r \partial \theta} \)