Hints for Review Sheet:

1. The limit exists & there's only really
   one sure way to prove that.

2. There's a solid, and I didn't state it clearly, but
   you're finding \( \iiint \vec{F} \cdot d\vec{S} \).

   You want the divergence theorem.

3. It's a solid again.

   Div. Thm

4. Fundamental Theorem of Line Integrals (FTLI).

   \[ f = x \sin y + y \cos z \]

5. \( f(xy^2) = xy^2 \), \( g = 2xy + 2xz + 2y^2 \)

6. Let \( u \) & \( v \) be what you're tempted to make them,
   that is, \( v = y + x \), \( u = y - x \). Solve for \( x \) and \( y \).

   The bounds in \( u \) & \( v \) don't make a rectangle in this
   problem. They give
4. Start with the right hand side, and keep in mind that it's equivalent to
\[ e^{2s} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = \frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2}. \]

8. The cross section in the \( yz \)-plane is
\[ z^2 = 3y^2 \]
\[ or \]
\[ z = \pm \sqrt{3}y \]

We want to use spherical coordinates, with a fixed \( \phi \)
(since \( \rho \) & \( \theta \) are not fixed) in the parametrization.

9. R:

With just the transformation, we don't have the integral to give a hint.
Still, \( y = \frac{1}{x} \) and \( y = \frac{4}{x} \) hint strongly at isolating the 1 & 4:
\[ y \cdot x = 1 \quad \& \quad y \cdot x = 4. \]
So let \( u = y \cdot x \)

Γ No for \( y = x, \ y = 4x \). Rewrite them as
\[ \& \ y = 1 \cdot x, \ y = 4 \cdot x \]
& isolate the constants again.
10. Define \( \vec{F} = \langle F_1, F_2, F_3 \rangle \), \( \vec{G} = \langle G_1, G_2, G_3 \rangle \).

Take one side and one operation at a time.

11. \( \vec{E} \). This can be done as a straightforward integral, or using Stoke's Thm.

Actually, applying it in one direction and then backwards for a different surface with the same boundary is the best, with the easiest (and fewest) integration.

12. \( g(x,y,z) = C_1 \), \( h(x,y,z) = C_2 \).

\[ \nabla f = \lambda \nabla g + \mu \nabla h. \]

13. Use chain rule.

14. When you find curl \( \vec{F} \), plug in the parametrization of the sphere. Note how this interacts with \( \vec{n} \) on the unit sphere (b.t.w. \( \vec{n} = \langle x, y, z \rangle \) on the unit sphere).

15. Break it up into 4ths
16. This one should be pretty straight-forward:

\[ \iiint \vec{F} \cdot d\vec{S} \]

![Diagram with vector field and surface integral](image)

\[ \langle \cos \Theta, \sin \Theta, -4 \rangle \]

Oops... 2nd 16: \( f = xyz, \ g = x + 2y + 3z = 6. \)

![Diagram with coordinate axes](image)

17. (a) \( \frac{\partial z}{\partial x} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial r} \)

Similarly for (b), only \( \frac{\partial x}{\partial r} \) & \( \frac{\partial x}{\partial \theta} \), \( \frac{\partial y}{\partial r} \) & \( \frac{\partial y}{\partial \theta} \) must be filled in.

(c) \( \frac{\partial^2}{\partial r^2} [F] = \frac{\partial F}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial r} \), no matter what \( F \) is (even if it's already a derivative).