Notes on finding $f$, such that $\mathbf{F} = \nabla f$ when $\mathbf{F}$ is conservative

28 Apr 2014

Notes:

So, basically, $f$ is a function that has some pieces involving $x$, $y$, and $z$ in all sorts of combinations:

- only $x$,
- only $y$,
- $x$ and $y$,
- $x$ and $z$,
- all 3,
- etc.

When we take the partial derivatives of these, if a piece involves $y$ and $z$, but not $x$, then it will show up in the 2nd and 3rd components of $\nabla f = \mathbf{F}$, or $f_y$ and $f_z$. It only existed once in the original function, so we only count it the once.

I’m sure most of you are having no trouble only counting things when they only appear in one integral. As an example, let’s say $e^x$ appeared in $f$, with no $y$ or $z$ involved. Then, when we took $\int (f_x) dx$, we’d get $e^x$ popping up exactly once there, but not in $\int (f_y) dy$ or $\int (f_z) dz$.

Let’s say $f$ had a component of $y \cdot \sin(z)$. Then, this would show up as $\sin(z)$ in $f_y$ (note that $f_y$ might have other parts (components), but we’re only focusing on this one for now). It would look like $y \cdot \cos(z)$ in $f_z$. So, when we took our integrals, it would pop up twice:

- not at all in $\int (f_x) dx$ (it had no $x$ parts),
- exactly once in $\int (f_y) dy$,
- and exactly once in $\int (f_z) dz$.

It’s tempting to gather all the pieces and add them together, but this will result in double and triple times the stuff that was actually originally in $f$.

You can always check your work by working backwards with your resulting $f$: take the partial derivatives.

A Game:

If you’re studying with a friend, I actually encourage you to make up your own problems for each other. It’s really easy.

1. Write down 2 or 3 functions $f(x, y, z)$.
2. Take their gradients.
3. Trade.

4. Prove your partner’s (or partners’) functions are conservative, and find the original functions.

Examples (with answers on the next page):

1. $\mathbf{F}_1 = \langle 2xyz, x^2z - e^y \sin(z), x^2y - e^y \cos(z) \rangle$

2. $\mathbf{F}_2 = \langle 2xe^y + \frac{y}{z} \sin(xy), x^2e^y + 2 \ln(z) + \frac{x}{z} \sin(xy), \frac{2y}{z} + \frac{\cos(xy)}{z^2} \rangle$
Answers:

1. $f = x^2yz - e^y \sin(z)$
2. $f = x^2e^y + 2y \ln(z) - \frac{\cos(xy)}{z}$