Math 2374, Lecture 10: Quiz 5
20 October 2011

Name: ___________________________  Section #: ______________________

For all questions, show your work. State formulas, if they are needed. Use the back of this sheet if you need more space.

1. Imagine a parabolic prism (a tall, solid object, whose cross section is a part of a parabola) defined on the $xy$–plane by $y = 0$ and $y = x^2 - 4$. This object is then cut off by the planes $z = 0$ and $z = 5 + x$ (so that the top of the object is angled, with one side of the flat edge being of height 3 and the other of height 5). Find the volume of this figure. 

**Hints:** 1. Draw the cross sections in the $xz$– and $xy$– planes; 2. This is very similar to the problems from lab 3.

**Answer:** We set up the integral like we did in lab: start with the “height” differences, and integrate with respect to $z$. Since the equations were given in the problem, as well as what values $z$ will take on the endpoints, we have an easy time of picking the limits of integration. This makes the innermost integral. Then, we use the cross section in the $xy$–plane to find an equation for $y$ in terms of $x$. These become the limits of integration for the integration with respect to $y$. Finally, we use the cross $xy$– plane cross section once again to obtain the absolute smallest and largest values $x$ can achieve ($–2$ and 2, respectively). We obtain the integral:

$$
\int_{–2}^{2} \int_{x^2 – 4}^{0} \int_{0}^{5+x} dz \ dy \ dx
$$

$$
= \int_{–2}^{2} \int_{x^2 – 4}^{0} (5 + x)dy \ dx
$$

$$
= \int_{–2}^{2} (5 + x)y|_{x^2 – 4}^{0} dx
$$

$$
= \int_{–2}^{2} (5 + x)(4 – x^2)dx
$$

$$
= \int_{–2}^{2} (20 + 4x – 5x^2 – x^3)dx
$$

$$
=(20x + 2x^2 – 5x^3/3 – x^4/4)|_{–2}^{2}
$$

$$
=20(2 – (–2)) + 2(4 – 4) – 5/3 · (8 – (–8)) – 1/4 · (16 – 16)
$$

$$
=80 – 80/3 = 160/3
$$

2. If $\vec{c}(t) = < \cos(t^2), 2 \cdot t^2, \sin(t^2) >$, then what is the length of the line ($L(t)$) as $3 \leq t \leq 6$?

**Answer:** We solve this problem through a few steps:

i) $\vec{c}'(t) = < -2t \sin(t^2), 4t, 2t \cos(t^2) >$
ii) 

\[ ||\mathbf{c}'(t)|| = \sqrt{(-2t)^2 \sin^2(t^2) + (4t)^2 + (2t)^2 \cos^2(t^2)} \]

\[ = \sqrt{4t^2 + 16t^2 + 20t^2} \]

\[ = 2\sqrt{5}t \]

iii) Now, we make use of the bounds on \( t \) and use them as limits of integration:

\[ L(t) = \int_{3}^{6} ||\mathbf{c}'(t)|| \, dt \]

\[ = \int_{3}^{6} (2\sqrt{5}t) \, dt \]

\[ = \sqrt{5}t^2 \bigg|_{3}^{6} \]

\[ = \sqrt{5}(36 - 9) = 27\sqrt{5} \]