2. Mark each statement true or false. Justify your answer.

(b) The only case where \( p \implies q \) is false is when \( p \) is true and \( q \) is false.

\[
\begin{array}{cc|c|c}
P & q & P \implies q & True\
\hline
T & T & T & T\
T & F & F & T\
F & T & T & T\
F & F & T & T\
\end{array}
\]

(c) "If \( p \), then \( q \)" is equivalent to "\( p \) whenever \( q \)."

\[
\begin{array}{cc|c|c}
P & q & if \ p \ then \ q & p \ whenever \ q\
\hline
T & T & T & T\
T & F & F & T\
F & T & T & T\
F & F & T & T\
\end{array}
\]

These do not match.

False; "\( p \) whenever \( q \)" is the same as "if \( q \), then \( p \)."

(d) The negation of \( p \implies q \) is \( q \implies p \).

False: Compare the truth tables for \( \neg(p \implies q) \) and \( q \implies p \).

\[
\begin{array}{cc|c|c|c|c}
P & q & \neg(p \implies q) & q \implies p & True\
\hline
T & T & F & T & T\
T & F & T & T & T\
F & T & T & T & T\
F & F & T & T & T\
\end{array}
\]

These do not match.
4. Write the negation of each statement.
   
   (a) The function \( f(x) = x^2 - 9 \) is continuous at \( x = 3 \).
   
   The function \( f(x) = x^2 - 9 \) is not continuous at \( x = 3 \).
   
   (it helps to only think of the subject and verb).

   (c) Four and nine are relatively prime.
   
   The only thing we can negate is "are".
   
   Two objective nouns
   
   The "and" doesn't connect 2 clauses/statements.
   
   It connects two nouns.

   Four and nine are not relatively prime.
   
   (Note: the original sentence was true; the negation is false always).

   (d) \( x \) is in \( A \), or \( x \) is not in \( B \) \( \iff p \lor q \)
   
   This is a full statement

   Negate: \( \neg (p \lor q) \iff [\neg p \land \neg q] \). Note that \( \neg q \) is the negation of "\( x \) is not in \( B \)." So, \( \neg q \) is "\( x \) is in \( B \)."

   \( \neg p \land \neg q \iff \text{ } x \text{ is not in } A \text{ and } x \text{ is in } B \)
4 (e) If $x < \frac{7}{9}$, then $f(x)$ is not in $C$. 

$$p \implies \neg q$$

$$(\text{If } p, \text{ then } q) \iff (p \implies q).$$

We must negate an implication.

$$\neg(p \implies q) \iff [p \land \neg q]$$

The result:

$$\neg q \text{ and } f(x) \text{ is in } C.$$ 

8. Construct a truth table for each statement.

(a) $p \quad q \quad \neg q \quad p \lor \neg q$

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
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<tr>
<td>T</td>
<td>F</td>
<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
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</tbody>
</table>

(b) $p \quad \neg p \quad p \land \neg p$ (should be false, always).

<p>| | | |</p>
<table>
<thead>
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<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

(c) $[\neg q \land (p \implies q)] \implies \neg p$

$$p \quad q \quad [\neg q \land (p \implies q)] \implies \neg p$$

<p>| | | | |</p>
<table>
<thead>
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</thead>
<tbody>
<tr>
<td>T</td>
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<td>F</td>
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<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>
9. Indicate whether each statement is true or false.

(a) \(3 \leq 5\) and \(11\) is odd
\[\begin{array}{c|c|c}
\hline
T & T & T \\
\hline
\end{array}\]

\[\text{True.}\]

(b) \(3^2 = 8\) or \(2 + 3 = 5\)
\[\begin{array}{c|c|c}
\hline
F & T & T \\
\hline
\end{array}\]

\[\text{True}\]

(c) \(5 > 8\) or \(3\) is even
\[\begin{array}{c|c|c}
\hline
F & F & T \\
\hline
\end{array}\]

\[\text{False}\]

(d) If \(6\) is even, then \(9\) is odd.
\[\begin{array}{c|c|c|c}
\hline
T & T & T \\
\hline
\end{array}\]

\[\text{True}\]

(e) If \(8 < 3\), then \(2^2 = 5\)
\[\begin{array}{c|c|c|c}
\hline
F & T & T \\
\hline
\end{array}\]

\[\text{The hypothesis is false, so we don’t care that the conclusion is false. The implication is true.}\]

(f) If \(7\) is odd, then \(10\) is prime.
\[\begin{array}{c|c|c|c}
\hline
T & F & F \\
\hline
\end{array}\]

\[\text{False.}\]

(g) If \(8\) is even and \(5\) is not prime, then \(4 < 7\).
\[\begin{array}{c|c|c|c|c|c}
\hline
T & F & T & T & T \\
\hline
\end{array}\]

\[\text{The implication is true.}\]
9 (h) If \( \frac{3}{4} \) is odd or \( \frac{4}{6} \) then \( \frac{9}{r} \).

For this one, let's do a table. We'll only need one line. Note that "if \((p \lor q)\), then \(r\)" is the same as "\((p \lor q) \Rightarrow r\)"

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>((p \lor q) \Rightarrow r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

The implication fails, which is what we care about. False.

(i) If both \(5 - 3 = 2\) and \(5 + 3 = 2\), then \(9 = 4\).

The "both" isn't really necessary.

Truth table (with one line)

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>((p \land q) \Rightarrow r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

The conditional statement (or implication) is True.

(j) It is not the case that 5 is even or 7 is prime.

This is a negation: \(\neg \left( (5 \text{ is even}) \lor (7 \text{ is prime}) \right)\)

which is equivalent to

\[ \neg (5 \text{ is even}) \] and \[ \neg (7 \text{ is prime}) \]

or

5 is not even and 4 is not prime. False.
13. Define a new sentential connective \( \Diamond \), called "nor," by the following:

\[
\begin{array}{c|c|c}
P & q & P \Diamond q \\
\hline
T & T & F \\
T & F & F \\
F & T & F \\
F & F & T \\
\end{array}
\]

\( \text{only true when both } P \& q \text{ are false.} \)

(a) Use a truth table to show that \( P \Diamond P \) is logically equivalent to \( \sim P \).

\[
\begin{array}{c|c|c}
P & \sim P & P \Diamond P \\
\hline
T & F & F \\
F & T & T \\
\end{array}
\]

They are equivalent.

(b) Complete a truth table for \( (P \Diamond P) \Diamond (q \& q) \).

\[
\begin{array}{c|c|c|c|c}
P & q & (P \Diamond P) \Diamond (q \& q) & \text{or we could use} & (\sim P) \Diamond (\sim q) \\
\hline
T & T & F & T & F \\
T & F & F & F & F \\
F & T & F & F & T \\
F & F & T & T & T \\
\end{array}
\]

(c) Which of our basic connectives \( (P \& q, p \& q, p \Rightarrow q, p \Leftrightarrow q) \) is logically equivalent to \( (P \Diamond P) \Diamond (q \& q) \)?

\( P \& q \), for its truth table is

\[
\begin{array}{c|c|c}
P & q & P \& q \\
\hline
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\end{array}
\]