Limit Theorems

1. Find the limit of the sequence $a_n = \frac{(2n^2 + 100)}{(3n - n^2)}$, and justify in two ways: first directly and then using Theorem 1 from class yesterday, which appears as Theorem 4.1.8 in the textbook.

2. Suppose that $a_n \to a$. Show directly (that is, without using the product law, which we will prove Wednesday) that $a_n^2 \to a^2$. Hint: write $|a_n^2 - a^2|$ as the product of something that is bounded and something that becomes arbitrarily small.
3. For these statements, prove or give a counterexample, and assume for now the limit theorems for constant multiplication, multiplication, and division, as well as addition:

(a) If \((a_n)\) and \((b_n)\) diverge, then \((a_n + b_n)\) diverges.
(b) If \((a_n)\) and \((b_n)\) diverge, then \((a_n b_n)\) diverges.
(c) If \((a_n)\) and \((a_n + b_n)\) converge, then \((b_n)\) converges.
(d) If \((a_n)\) and \((a_n b_n)\) converge, then \((b_n)\) converges.