Practice using mathematical induction

1. (3.1 #11) Prove that for all natural numbers $n$, we have

$$
\sum_{i=1}^{n} \frac{i}{(i+1)!} = 1 - \frac{1}{(n+1)!}.
$$

2. (3.1 #19) Conjecture a formula for the sum $5 + 9 + 13 + \cdots + (4n + 1)$, and prove your conjecture using mathematical induction. (Write out a table of values for some small $n$; do you notice a pattern? Or, can you “fit” a simple function of $n$ to the first few values?)
3. (3.1 #22) Prove deMoivre’s formula: for all natural numbers \( n \), we have
\[
(cos x + i \sin x)^n = cos(nx) + i \sin(nx),
\]
where \( i^2 = -1 \). Use the identities
\[
\cos(a + b) = \cos a \cos b - \sin a \sin b
\]
and
\[
\sin(a + b) = \sin a \cos b + \cos a \sin b.
\]