Syllabus

Topics in Probability, Math 8660, Itô stochastic equations and elliptic and parabolic differential equations, Fall 2016

Lectures: 11:15-12:05 MWF VinH 1
Instructor: Nicolai Krylov, VinH 225, tel. 625-8338, nkrylov@umn.edu
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Office hours: MWF, 13:25-14:15
Textbook: Lecture notes will be provided

Final examination: Take home final due on December 22, 2016.

PREREQUISITE KNOWLEDGE: Basics of the theory of discrete time martingales.

APPROXIMATE OUTLINE OF THE COURSE: The course will be about the relations between diffusion processes (solutions of Itô stochastic equations) and second-order linear elliptic and parabolic differential equations. The goal is to show how probabilistic methods help obtain important information about solutions of PDEs.

A few homeworks will be assigned and will form part of the final grade.

The contents of the lecture notes is given on the next two pages (which have numbers i and ii).
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