Study guide for the final exam

Math 5485, Fall 2008

See sections 1-4 from study guide for the second midterm.

5. Eigenvalues and eigenvectors, continued (Chapter 4)
   (a) Reduction to symmetric tridiagonal form
      i. Why first reduce to tridiagonal before calculating all eigenvalues and eigenvectors
      ii. Properties of similarity transforms and orthogonal matrices,
      iii. How similarity transformation by a single Householder matrix put zeros in certain parts of matrix
      iv. How to combine these similarity transformations to turn into tridiagonal
      v. How eigenvectors are transformed in this process
   (b) Eigenvalues and eigenvectors of symmetric tridiagonal matrices
      i. The basic idea of how the QR algorithm works
      ii. Effect of pre- and post-multiplication by rotation matrix and how it underlies the QR algorithm
      iii. How eigenvectors are transformed in this process
      iv. Wilkinson shift will not be on exam
6. Interpolation (Chapter 5)
   (a) Basic ideas
      i. Interpolation versus approximation
      ii. Why polynomials are good choice in principle (Weierstrass approximation theorem)
      iii. With exception of solving tridiagonal system for cubic spline, be able to apply the interpolation scheme to word problems. Recognize if get bad results.
   (b) Lagrange form of interpolating polynomial
      i. Properties and formula of the Lagrange polynomials $L_{n,j}(x)$
      ii. Combining the $L_{n,j}$ to interpolate function values
      iii. Uniqueness of interpolating polynomial
      iv. Interpolation error
      v. Advantages and disadvantages of Lagrange form
   (c) Neville's algorithm
      i. What Neville's is good for
      ii. Applying Neville's algorithm to data
   (d) Newton's form of interpolating polynomial

i. What Newton's form is good for
ii. Divided differences
iii. Determining Newton's form from data

(e) Optimal points for interpolation
   i. Understand and use both definition and recurrence relation for Chebyshev polynomials
   ii. Understand relationship between Chebyshev polynomials and smallest monic polynomials
   iii. Understand and be able to apply implications of analysis of Chebyshev polynomials on choosing optimal interpolating points for maximum norm
   iv. Legendre polynomials and optimal interpolating points for Euclidean norm will not be on final exam

(f) Piecewise polynomial interpolation (in general)
   i. Motivations for using different lower-order polynomials on each subinterval
   ii. A partition

(g) Piecewise linear interpolation
   i. Definition of piecewise linear interpolant
   ii. Calculating piecewise linear interpolation
   iii. Error analysis

(h) Cubic spline interpolation
   i. Definition of cubic spline and how it maximize smoothness of piecewise cubic
   ii. Need for extra condition to solve for parameters
   iii. How one can calculate cubic spline efficiently (form tridiagonal system)
   iv. Differences among boundary conditions (not-a-knot, clamped, and natural)

(i) Hermite interpolation
   i. Properties of polynomials $H_i(x)$ and $\hat{H}_i(x)$
   ii. The Lagrange form of Hermite interpolating polynomial
   iii. The Newton form of Hermite interpolating polynomial
   iv. Calculating Hermite polynomial from data

(j) Hermite cubic interpolation
   i. Properties of Hermite cubic interpolant (sacrificing smoothness for matching derivative data)
   ii. Calculating Hermite cubic interpolant from data