Corrections to Second Printing (2008)

Last updated: August 30, 2017

*** Page xxii ***
Replace the last paragraph by:

We would also like to thank Nihat Bayhan, Joe Benson, Juan Cockburn, Richard Cook, Stephen DeSalvo, Anne Dougherty, Kathleen Fuller, Stuart Hastings, Jeffrey Humpherys, Roberta Jaskolski, Tian–Jun Li, James Meiss, Willard Miller, Jr., Timo Schürig, David Tieri, and Timothy Welle for sending us their comments, suggestions, and corrections to earlier printings of this book. A particular thanks to David Hiebeler for his careful reading and corrections.

*** Page xxii ***
Change Cheri Shakiban’s email address to cshakiban@stthomas.edu

*** Page 43 *** Theorem 1.29:
To avoid confusion, change “having the nonzero pivots on the diagonal” to “with nonzero diagonal entries”.

*** Page 43 *** Theorem 1.31:
Insert “with nonzero diagonal entries” after “diagonal”.

*** Page 51 *** second displayed equation:
Replace the summand $j$ by $j - 1$:

$$\sum_{j=1}^{n} (j - 1) = \frac{n^2 - n}{2}$$

*** Page 57 *** last displayed equation:
Replace 3210 by 32100:

$$10x + 1600y = 32100, \quad x + .6y = 22,$$
Replace 3210 by 32100:
\[
\begin{pmatrix}
1600 & 10 & 32100 \\
.6 & 1 & 22
\end{pmatrix}
\]

Exercise 2.3.39 (b):
Add closing bracket to \( W[f(x), g(x)] \).

Exercise 2.4.24 (b):
Change “Under the hypotheses of part (b)” to “Under the hypothesis of part (a)”.

Exercise 2.4.24 (b):
Change “Solving the homogeneous system \( \hat{U} y = 0 \), we conclude that” to
The two nonzero rows of \( \hat{U} \) form a basis for \( \text{corng} \ A^T \), and therefore

Exercise 3.2.31 (a):
Add \( T \) superscripts to \( (1, 2, 3)^T \) and \( (1, -1, 2)^T \).

Formula before Proposition 3.34:
Change \( dt \) to \( dx \).

Two lines before Proposition 3.34:
Change Theorem 3.31 to Theorem 3.28.

Exercise 3.4.22 (c):
Change “null vectors” to “null directions”.

Exercise 3.4.32:
Change “null vector” to “null direction” and \( K = A^T A \) to \( K = A^T C A \):
Show that \( 0 \neq z \) is a null direction for the quadratic form \( q(x) = x^T K x \) based on the Gram matrix \( K = A^T C A \) if and only if \( z \in \ker K \).
Exercise 3.4.35 (c):

Rephrase for clarity:
Show that $K$ is also a Gram matrix, by finding a matrix $A$ such that $K = A^T C A$.

Sentence after that containing (3.70):

Rephrase for clarity:
Note that $M$ is a lower triangular matrix with all positive diagonal entries, namely the square roots of the pivots: $m_{ii} = \sqrt{d_i}$.

Exercise 3.6.29:
Delete part (e). (“Orthogonal” and “orthonormal” are not yet defined.)

Exercise 3.6.51:
For each of the following . . .

Formula in middle of page:
$y^*$ and $z^*$ should not be bold face.

Theorem 4.4:
Delete the words “null vector”.

Equation (4.26):
The last equality, $c = \| b \|^2$, is correct provided one uses the weighted norm. However, to avoid confusion with the Euclidean norm used in (4.25), it would be better to write this as $c = b^T C b$.

Equation (4.30):
$$\| A x^* - b \| = \sqrt{\| b \|^2 - f^T x^*} = \sqrt{\| b \|^2 - b^T A (A^T A)^{-1} A^T b}.$$ 

Line after Equation (4.43):
Delete first “coefficient”:
The $m \times (n + 1)$ coefficient matrix . . .

Displayed equation in Proof of Theorem 4.16:
Middle term should be $y_k L_k(t_k)$:
$$p(t_k) = y_1 L_1(t_k) + \cdots + y_k L_k(t_k) + \cdots + y_{n+1} L_{n+1}(t_k) = y_k,$$
Exercise 5.1.11:
\[ \mathbf{u}_2 = \pm \left( \begin{array}{c} -\sin \theta \\ \cos \theta \end{array} \right) \]

Second line:
Replace “For exercises #1–8 use ” by “For Exercises #1–7 use ”.

First line under Figure:
Change “\(\mathbf{x}\) and \(\mathbf{y}\)” to “\(\mathbf{v}\) and \(\mathbf{w}\)”.

Remark:
\[ \ldots \text{solving the homogeneous adjoint system}, \ldots \]

Exercise 5.6.20 (c):
Change the sign in front of \(4x_3\) in last equation:
\[ x_1 + 2x_2 + 3x_3 = b_1, \quad x_2 + 2x_3 = b_2, \quad 3x_1 + 5x_2 + 7x_3 = b_3, \quad -2x_1 + x_2 + 4x_3 = b_4; \]

Equation (5.90):
Change \(e^{ikx_n}\) to \(e^{ikx_{n-1}}\) in first line.

Line -5:
Change \(n = 2^8 = 256\) to \(n = 2^9 = 512\).

Equation (6.9):
Insert space between 1 and \(-1\) in last row of matrix.

Two lines before (6.15):
Change \(K\mathbf{x} = \mathbf{f}\) to \(K\mathbf{u} = \mathbf{f}\).

Four lines after (6.15):
Change \(\mathbf{y} = A^{-1}\mathbf{f}\) to \(\mathbf{y} = A^{-T}\mathbf{f}\).

Exercise 6.2.1 (b):
Change last row of matrix:
\[ \begin{pmatrix} 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix} \]
Exercise 6.2.2:

Add labels to the wires:

Exercise 6.2.12 (a):

Start the exercise with:

Assuming all wires have unit resistance, find the voltage . . .

Line 19:
Delete “the” before “Section 6.1”.

Displayed equation above (6.51):

Change 0 to 0.

Equation (6.58):

Change the last formula to

\[ z_3 \cdot f = -\frac{\sqrt{3}}{2} f_1 + \frac{1}{2} g_1 + g_2 = 0. \]

Last line:

Change “first node” to “third node”.

Two lines before displayed equation for \( A^{**} \):

This serves to also eliminate . . .

Figure 6.13:

Label the bars in the figure:
Figure 6.16:
Label the bars in the figure:

Figure 6.17:
Label the bars in the figure:

Page 325  Line -5:
Change “three bars” to “five bars”.

Page 339  Equation (7.12):
Add period at end of equation.

Page 377  Definition 7.46 is incomplete. Here is a corrected version::

**Definition 7.46.** A complex vector space $V$ is called **conjugated** if it admits an operation of complex conjugation taking $u \in V$ to $\overline{u} \in V$ with the following properties: (a) conjugating twice returns one to the original vector: $\overline{\overline{u}} = u$; (b) compatibility with vector addition: $\overline{u + v} = \overline{u} + \overline{v}$; (c) compatibility with scalar multiplication: $\overline{\lambda u} = \overline{\lambda} \overline{u}$, for all $\lambda \in \mathbb{C}$ and $u, v \in V$.

Page 384  Exercise 7.5.1 (b):
$$\langle v, w \rangle = 2v_1 w_1 + 3v_2 w_2$$

Page 399  Final displayed equation:
The bar on the second term should extend over both $A$ and $v$:
$$\overline{A} \overline{v} = \overline{A} \overline{v} = \overline{\lambda v} = \overline{\lambda} \overline{v}.$$

Page 400  Two lines before Remark:
Change “combinations of the real eigenvalues” to “combinations of the real eigenvectors”.

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*** Page 411 *** Last line:
Delete “the” before “Section 8.6”.

*** Page 432 *** Fourth displayed formula. Switch $T$ superscript:
\[
A^+ = Q \Sigma^{-1} P^T = \begin{pmatrix}
.2444 & .1333 & .0556 & .1889 \\
.1556 & -.0667 & .1111 & .0444 \\
-.1111 & 0 & -.0556 & -.0556
\end{pmatrix}.
\]

*** Page 448 *** Equation (9.8) and line 19:
Change $\dot{u} = Au$ to $\dot{u} = Au$.

*** Page 455 *** Exercise 9.1.22:
Change $\dot{u} = Au$ to $\dot{u} = Au$.

*** Page 458 *** Theorem 9.13:
Change $\dot{u} = Au$ to $\dot{u} = Au$.

*** Page 465 *** Exercise 9.2.18:
Change $\dot{u} = -\nabla H$ to $\dot{u} = -\nabla H$.

*** Page 477 *** Exercise 9.4.15:
Change $v = A^T v$ to $\dot{v} = A^T v$.

*** Page 481 *** Exercise 9.4.34:
Change $\dot{u} = Au + e^{\lambda t} v$ to $\dot{u} = Au + e^{\lambda t} v$.

*** Page 483 *** Equation (9.55):
Correct final formula:
\[
e^{tA_2} = \begin{pmatrix}
\cos t & -\sin t & 0 \\
\sin t & \cos t & 0 \\
0 & 0 & 1
\end{pmatrix}.
\]

*** Page 488 *** line after (9.70):
Change $r_i > 0$ to $r_i \geq 0$.

*** Page 572 *** Displayed formula before (10.102):
The subscripts on $R$ and $Q$ are wrong: $A_2 = R_1 Q_1$.  

Change equations (10.106) and (10.107) to:

\[ A^k = (Q_0 Q_1 \cdots Q_{k-1}) \left( R_{k-1} \cdots R_1 R_0 \right). \]  
(10.106)

\[ S_k = Q_0 Q_1 \cdots Q_{k-1} = S_{k-1} Q_{k-1}, \]
\[ P_k = R_{k-1} \cdots R_1 R_0 = R_{k-1} P_{k-1}. \]  
(10.107)

Replace the paragraph after Theorem 10.57 by the following:

The last remaining item is a proof of Lemma 10.56. We write

\[ S = (u_1 \ u_2 \ \cdots \ u_n), \quad S_k = (u^{(k)}_1, \ldots, u^{(k)}_n) \]
in columnar form. Let \( t_{ij}^{(k)} \) denote the entries of the positive upper triangular matrix \( T_k \).

The first column of the limiting equation \( S_k T_k \to S \) reads

\[ t_{11}^{(k)} u^{(k)}_1 \to u_1. \]

Since both \( u^{(k)}_1 \) and \( u_1 \) are unit vectors, and \( t_{11}^{(k)} > 0 \),

\[ \| t_{11}^{(k)} u^{(k)}_1 \| = t_{11}^{(k)} \to \| u_1 \| = 1, \quad \text{and hence} \quad u^{(k)}_1 \to u_1. \]

The second column reads

\[ t_{12}^{(k)} u^{(k)}_1 + t_{22}^{(k)} u^{(k)}_2 \to u_2. \]

Taking the inner product with \( u^{(k)}_1 \to u_1 \) and using orthonormality, we deduce \( t_{12}^{(k)} \to 0 \), and so \( t_{22}^{(k)} u^{(k)}_2 \to u_2 \), which, by the previous reasoning, implies \( t_{22}^{(k)} \to 1 \) and \( u^{(k)}_2 \to u_2 \).

The proof is completed by working in order through the remaining columns, employing a similar argument at each step. Details are left to the interested reader.

Equation (11.21):

Insert minus sign before integral:

\[ u'(\ell) = -\int_0^\ell f(x) \, dx = 0, \]  
(11.21)

Line before (11.28):

Change “to satisfy” to “satisfy”.

3 lines after (11.40):

Change \( L[u] = u(y) \) to \( L_y[u] = u(y) \).

Equation (11.60):

Missing factor of \( c \) in differential equation:

\[ -cu'' = f(x), \quad u(0) = 0 = u(1), \]
Equation (11.59):

The middle expression is missing a $c$ in the denominator:

$$G(x, y) = \frac{(1 - y)x - \rho(x - y)}{c} = \begin{cases} 
  x(1 - y)/c, & x \leq y, \\
  y(1 - x)/c, & x \geq y, 
\end{cases}$$  \hspace{1cm} (11.59)

Two missing factors of $c$:

$$c \frac{du}{dx} = (1 - x)xf(x) + \int_0^x [-yf(y)]\,dy - x(1 - x)f(x) + \int_x^1 (1 - y)f(y)\,dy$$

$$= -\int_0^1 yf(y)\,dy + \int_x^1 f(y)\,dy.$$ 

Differentiating again, we conclude that $c \frac{d^2u}{dx^2} = -f(x)$, as claimed.

Delete “is” after $K = L^* \circ L$.

Change “Chapter 4.1” to “Section 4.1”.

Solution 1.2.4 (d):

$$A = \begin{pmatrix}
  2 & -1 & 2 \\
  -1 & -1 & 3 \\
  3 & 0 & -2
\end{pmatrix}, \quad x = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

Solution 1.2.4 (f):

$$b = \begin{pmatrix}
  -3 \\
  -5 \\
  2 \\
  1
\end{pmatrix}.$$ 

Solution 1.4.15 (a):

$$\begin{pmatrix}
  0 & 0 & 1 \\
  0 & 1 & 0 \\
  1 & 0 & 0
\end{pmatrix}.$$ 

Solution 1.8.4:

(i) $a \neq b$ and $b \neq 0$; (ii) $a = b \neq 0$, or $a = -2$, $b = 0$; (iii) $a \neq -2$, $b = 0$. 

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Solution 1.8.23 (e):
\[(0,0,0)^T;\]

Solution 2.5.5 (b):
\[\mathbf{x}^* = (1,-1,0)^T, \quad \mathbf{z} = z \left(-\frac{2}{7}, -\frac{1}{7}, 1\right)^T;\]

Solution 3.4.22 (v):
Change “null vectors” to “null directions”.

Solution 3.4.32:
Change all $\mathbf{x}$’s to $\mathbf{z}$:
\[0 = \mathbf{z}^T K \mathbf{z} = \mathbf{z}^T A^T C A \mathbf{z} = \mathbf{y}^T C \mathbf{y}, \text{ where } \mathbf{y} = A \mathbf{z}. \text{ Since } C > 0, \text{ this implies } \mathbf{y} = 0, \text{ and hence } \mathbf{z} \in \ker A = \ker K.\]

Solution 4.4.27 (a):
Change “the interpolating polynomial” to “an interpolating polynomial”.

Solution 4.4.52 (b):
Delete the sentence:
(The solution given is for the square $S = \{0 \leq x \leq 1, \ 0 \leq y \leq 1\}$.)

Solution 5.1.14 (a):
\[v_2 = \pm \left(-\sin \theta, \frac{1}{\sqrt{2}} \cos \theta\right)^T\]

Solution 5.4.15:
\[p_0(x) = 1, \quad p_1(x) = x, \quad p_2(x) = x^2 - \frac{1}{3}, \quad p_3(x) = x^3 - \frac{9}{10} x.\]
(The solution given in the text is for the interval $[0,1]$, not $[-1,1]$.)

Solution 5.5.6 (ii) (c):
\[\left(\frac{23}{43}, \frac{19}{43}, -\frac{1}{43}\right)^T.\]
Solution 6.2.1 (b) The solution corresponds to the revised exercise — see correction on page 311.

For the given matrix, the solution is

Solution 6.2.12 & 6.2.13:
Change all e’s to y’s.

Solution 6.3.5 (b):
\[
\begin{align*}
\frac{3}{2} u_1 - \frac{1}{2} v_1 - u_2 &= f_1, \\
-\frac{1}{2} u_1 + \frac{3}{2} v_1 &= g_1, \\
-u_1 + \frac{3}{2} u_2 + \frac{1}{2} v_2 &= f_2, \\
\frac{1}{2} u_2 + \frac{3}{2} v_2 &= g_2.
\end{align*}
\]

Solution 8.5.26:
Change (b) to (c).

Solution 9.1.28 (g):
Change \( u \) to \( \hat{u} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} u \) to \( \hat{u} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} u \).

Solution 11.2.8 (d):
\[
f'(x) = 4 \delta(x + 2) + 4 \delta(x - 2) + \begin{cases} 1, & |x| > 2, \\ -1, & |x| < 2, \end{cases}
\]
\[
f''(x) = 4 \delta'(x + 2) + 4 \delta'(x - 2) - 2 \delta(x + 2) + 2 \delta(x - 2).
\]

Solution 11.2.31 (a):
\[
u_n(x) = \begin{cases} x(1 - y), & 0 \leq x \leq y - \frac{1}{n}, \\ -\frac{1}{4} n x^2 + \left( \frac{1}{2} n - 1 \right) x y - \frac{1}{4} n y^2 + \frac{1}{2} y + \frac{1}{2} x - \frac{1}{4 n}, & |x - y| \leq \frac{1}{n}, \\ y(1 - x), & y + \frac{1}{n} \leq x \leq 1. \end{cases}
\]
Solution 11.3.3 (c):

(i) \( u_*(x) = \frac{1}{2}x^2 - \frac{5}{2} + x^{-1} \),

(ii) \( \mathcal{P}[u] = \int_1^2 \left[ \frac{1}{2}x^2 (u')^2 + 3x^2 u \right] dx, \quad u'(1) = u(2) = 0, \)

(iii) \( \mathcal{P}[u_*] = -\frac{37}{20} = -1.85 \),

(iv) \( \mathcal{P}[x^2 - 2x] = -\frac{11}{6} = -1.83333, \quad \mathcal{P}[-\sin \frac{1}{2} \pi x] = -1.84534. \)

Solution 11.5.7 (b):

\[
\lambda = -\omega^2 < 0, \quad G(x, y) = \begin{cases} 
\sinh \omega (y - 1) \sinh \omega x & , \quad x < y, \\
\frac{\omega \sinh \omega}{\sinh \omega (x - 1) \sinh \omega y} & , \quad x > y;
\end{cases}
\]

\[
\lambda = 0, \quad G(x, y) = \begin{cases} 
x(y - 1) , & \quad x < y, \\
y(x - 1) , & \quad x > y;
\end{cases}
\]

\[
\lambda = \omega^2 \neq n^2 \pi^2 > 0, \quad G(x, y) = \begin{cases} 
\sin \omega (y - 1) \sin \omega x & , \quad x < y, \\
\frac{\omega \sin \omega}{\sin \omega (x - 1) \sin \omega y} & , \quad x > y.
\end{cases}
\]