Replace the last paragraph by:

We would also like to thank Nihat Bayhan, Joe Benson, Juan Cockburn, Richard Cook, Stephen DeSalvo, Anne Dougherty, Kathleen Fuller, Mary Halloran, Stuart Hastings, Jeffrey Humpherys, Roberta Jaskolski, Tian-Jun Li, James Meiss, Willard Miller, Jr., Sean Rostami, Timo Schürg, David Tieri, and Timothy Welle for sending us their comments, suggestions, and corrections to earlier printings of this book. A particular thanks to David Hiebeler for his careful reading and corrections.

Change Cheri Shakiban’s email address to cshakiban@stthomas.edu

To avoid confusion, change “having the nonzero pivots on the diagonal” to “with nonzero diagonal entries”.

Insert “with nonzero diagonal entries” after “diagonal”.

Replace the summand $j$ by $j - 1$:

$$\sum_{j=1}^{n} (j - 1) = \frac{n^2 - n}{2}$$

Replace 3210 by 32100:

$$10x + 1600y = 32100, \quad x + .6y = 22,$$
 Replace $3210$ by $32100$:

\[
\begin{pmatrix}
1600 & 10 \\
.6 & 1
\end{pmatrix}
\begin{pmatrix}
32100 \\
22
\end{pmatrix}
\]

Correct last entry in third and fourth column vectors:

\[
\begin{pmatrix}
0 \\
1 \\
-1
\end{pmatrix}
= c_1 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}
= \begin{pmatrix} c_1 + 2 c_2 \\ -2 c_1 - 3 c_2 \\ c_1 + c_2 \end{pmatrix}
\]

Correct third equation:

\[
c_1 + 2 c_2 = 0, \quad -2 c_1 - 3 c_2 = 1, \quad c_1 + c_2 = -1.
\]

Add closing bracket to $W[f(x), g(x)]$.

Change “Under the hypotheses of part (b)” to “Under the hypothesis of part (a)”.

The two nonzero rows of $\widehat{U}$ form a basis for corng $A^T$, and therefore

Delete first “both”s:

Inner products and norms lie at the heart of linear (and nonlinear) analysis, in both finite-dimensional vector spaces and infinite-dimensional function spaces. It is impossible to overemphasize their importance for theoretical developments, practical applications, and the design of numerical solution algorithms.

Add $^T$ superscripts to $(1,2,3)^T$ and $(1,-1,2)^T$.

Change $dt$ to $dx$. 
Two lines before Proposition 3.34:
Change Theorem 3.31 to Theorem 3.28.

Exerci...e 3.4.22 (c):
Change “null vectors” to “null directions”.

Exercise 3.4.32:
Change “null vector” to “null direction” and $K = A^T A$ to $K = A^T C A$:
Show that $0 \neq z$ is a null direction for the quadratic form $q(x) = x^T K x$ based on the Gram matrix $K = A^T C A$ if and only if $z \in \ker K$.

Exercise 3.4.35 (c):
Rephrase for clarity:
Show that $K$ is also a Gram matrix, by finding a matrix $A$ such that $K = A^T C A$.

Sentence after that containing (3.70):
Rephrase for clarity:
Note that $M$ is a lower triangular matrix with all positive diagonal entries, namely the square roots of the pivots: $m_{ii} = \sqrt{d_i}$.

Exercise 3.6.29:
Delete part (e). (“Orthogonal” and “orthonormal” are not yet defined.)

Exercise 3.6.51:
For each of the following . . .

Formula in middle of page:
y* and z* should not be bold face.

Theorem 4.4:
delete the words “null vector”.

Equation (4.26):
The last equality, $c = \| b \|^2$, is correct provided one uses the weighted norm. However, to avoid confusion with the Euclidean norm used in (4.25), it would be better to write this as $c = b^T C b$.

Equation (4.30):
$$\| A x^* - b \| = \sqrt{\| b \|^2 - f^T x^*} = \sqrt{\| b \|^2 - b^T A (A^T A)^{-1} A^T b}.$$
Line after Equation (4.43):

Delete first “coefficient”:
The $m \times (n + 1)$ coefficient matrix . . .

Page 204

Displayed equation in Proof of Theorem 4.16:

Middle term should be $y_k L_k(t_k)$:

\[ p(t_k) = y_1 L_1(t_k) + \cdots + y_k L_k(t_k) + \cdots + y_{n+1} L_{n+1}(t_k) = y_k, \]

Page 221

Exercise 5.1.11:

\[ u_2 = \pm \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix} \]

Page 231

Second line:

Replace “For exercises #1–8 use ” by “For Exercises #1–7 use ”.

Page 245

First line under Figure:

Change “x and y” to “v and w”.

Page 274

Remark:

. . . solving the homogeneous adjoint system, . . .

Page 276

Exercise 5.6.20 (c):

Change the sign in front of $4x_3$ in last equation:

\[ x_1 + 2x_2 + 3x_3 = b_1, \quad x_2 + 2x_3 = b_2, \quad 3x_1 + 5x_2 + 7x_3 = b_3, \quad -2x_1 + x_2 + 4x_3 = b_4; \]

Page 279

Equation (5.90):

Change $e^{ikx_n}$ to $e^{ikx_{n-1}}$ in first line.

Page 283

Figure 5.13:

Change $x^2 - 2\pi x$ to $2\pi x - x^2$.

Page 284

Figure 5.14:

Change $x^2 - 2\pi x$ to $2\pi x - x^2$.

Page 285

Line -5:

Change $n = 2^8 = 256$ to $n = 2^9 = 512$.

Page 296

Equation (6.9):

Insert space between 1 and $-1$ in last row of matrix.
Two lines before (6.15):
Change $Kx = f$ to $Ku = f$.

Four lines after (6.15):
Change $y = A^{-1}f$ to $y = A^{-T}f$.

Exercise 6.2.1 (b):
Change last row of matrix:
$$
\begin{pmatrix}
0 & 0 & 1 & -1 \\
1 & 0 & 0 & -1 \\
0 & -1 & 1 & 0 \\
1 & 0 & -1 & 0 \\
\end{pmatrix}
$$

Exercise 6.2.2:
Add labels to the wires:

Exercise 6.2.12 (a):
Start the exercise with:
Assuming all wires have unit resistance, find the voltage . . .

Line 19:
Delete “the” before “Section 6.1”.

Displayed equation above (6.51):
Change 0 to 0.

Equation (6.58):
Change the last formula to
$$
z_3 \cdot f = -\frac{\sqrt{3}}{2} f_1 + \frac{1}{2} g_1 + g_2 = 0.
$$

Last line:
Change “first node” to “third node”.

Two lines before displayed equation for $A^{**}$:
This serves to also eliminate . . .
Label the bars in the figure:

Figure 6.13:

Label the bars in the figure:

Figure 6.16:

Label the bars in the figure:

Figure 6.17:

Line -5:
Change “three bars” to “five bars”.

Equation (7.12):
Add period at end of equation.

Definition 7.46 is incomplete. Here is a corrected version::

**Definition 7.46.** A complex vector space \( V \) is called *conjugated* if it admits an operation of *complex conjugation* taking \( u \in V \) to \( \overline{u} \in V \) with the following properties:

(a) conjugating twice returns one to the original vector: \( \overline{\overline{u}} = u \); (b) compatibility with vector addition: \( \overline{u + v} = \overline{u} + \overline{v} \); (c) compatibility with scalar multiplication: \( \lambda \overline{u} = \overline{\lambda u} \), for all \( \lambda \in \mathbb{C} \) and \( u, v \in V \).

Exercise 7.5.1 (b):
\[
\langle v, w \rangle = 2v_1w_1 + 3v_2w_2
\]

Final displayed equation:
The bar on the second term should extend over both \( A \) and \( v \):
\[
\overline{A} \overline{v} = \overline{A v} = \overline{\lambda v} = \overline{\lambda v}.
\]
Two lines before Remark:
Change “combinations of the real eigenvalues” to “combinations of the real eigenvectors”.

Page 410 line 16:
Delete $T$ on formula for $S = (v_1, v_2, \ldots, v_n)$.

Last line:
Delete “the” before “Section 8.6”.

Fourth displayed formula. Switch $^T$ superscript:
\[
A^+ = Q \Sigma^{-1} P^T = \begin{pmatrix}
0.2444 & 0.1333 & 0.0556 & 0.1889 \\
0.1556 & -0.0667 & 0.1111 & 0.0444 \\
-0.1111 & 0 & 0.0556 & -0.0556
\end{pmatrix}
\]

Definition 8.46, first line:
Change $w_1, \ldots, w_j \in \mathbb{C}^m$ to $w_1, \ldots, w_j \in \mathbb{C}^n$.

line -2:
Change “Thus, $w_2$ a generalized . . .” to “Thus, $w_2$ is a generalized . . .”

Equation (9.8) and line 19:
Change $\dot{u} = Au$ to $\dot{u} = Au$.

Exercise 9.1.22:
Change $\dot{u} = Au$ to $\dot{u} = Au$.

Theorem 9.13:
Change $\dot{u} = Au$ to $\dot{u} = Au$.

Exercise 9.2.18:
Change $\dot{u} = -\nabla H$ to $\dot{u} = -\nabla H$.

Exercise 9.4.15:
Change $v = A^T v$ to $\dot{v} = A^T v$.

Exercise 9.4.34:
Change $\dot{u} = Au + e^{\lambda t} v$ to $\dot{u} = Au + e^{\lambda t} v$. 
### Page 483
Equation (9.55):
Correct final formula:
\[
e^{t A} z = \begin{pmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\]

### Page 488
line after (9.70):
Change \( r_i > 0 \) to \( r_i \geq 0 \).

### Page 501
Lines 2–4 after (9.96):
Switch “first” and “second”:
... the second, vibrating with frequency \( \omega \), represents the internal or natural vibrations of the system, while the first, with frequency \( \eta \), represents the response ...

### Page 523
Exercise 10.1.41:
Change \( x_0, x_1, \ldots \) to \( u^{(0)}, u^{(1)}, \ldots \)

### Page 526
Line 4:
Change \( \lambda_1 = -\frac{2}{3} \) to \( \lambda_1 = \frac{2}{3} \):

### Page 526
Line 5:
Change \(-\frac{2}{3}\) to \( \frac{2}{3} \):
\( \lambda_1 = \frac{2}{3} \)

### Page 526
Line 8:
Change “... the first ten iterates are” to “... iterates \( u^{(11)}, \ldots, u^{(20)} \) are”

### Page 564
Lines 15–17:
Change the parameter \( t_1 \) so that the corresponding residual vector
\[
r_1 = f - Ku_1 = r_0 - t_1 Kv_1
\]
is as close to \( \mathbf{0} \) (in the Euclidean norm) as possible. This occurs when \( r_1 \) is orthogonal to \( r_0 \) (why?), and so we require
\[
0 = r_0^T r_1 = \| r_0 \|^2 - t_1 r_0^T Kv_1 = \| r_0 \|^2 - t_1 \langle r_0, v_1 \rangle = \| r_0 \|^2 - t_1 \langle v_1, v_1 \rangle.
\]

The parameter \( t_1 \) that minimizes
\[
p(u_1) = p(t_1 v_1) = \frac{1}{2} t_1^2 v_1^T K v_1 - t_1 v_1^T f = \frac{1}{2} t_1^2 \langle v_1, v_1 \rangle - t_1 \| r_1 \|^2.
\]
Correct second and third terms in displayed formula:

\[ 0 = \langle \langle v_2, v_1 \rangle \rangle = \langle \langle r_1 + s_1 v_1, v_1 \rangle \rangle = \langle \langle r_1, v_1 \rangle \rangle + s_1 \langle \langle v_1, v_1 \rangle \rangle, \]

Delete “as small as possible, which is accomplished by requiring it to”

The subscripts on \( R \) and \( Q \) are wrong: \( A_2 = R_1 Q_1 \).

For each eigenvalue, the computation of the corresponding eigenvector can be most efficiently accomplished by applying the shifted inverse power method of Exercise 10.6.7 with parameter \( \mu \) chosen near the computed eigenvalue.

Change equations (10.106) and (10.107) to:

\[
A^k = (Q_0 Q_1 \cdots Q_{k-1}) (R_{k-1} \cdots R_1 R_0).
\]

\[
S_k = Q_0 Q_1 \cdots Q_{k-1} = S_{k-1} Q_{k-1};
\]

\[
P_k = R_{k-1} \cdots R_1 R_0 = R_{k-1} P_{k-1}.
\]

Replace the paragraph after Theorem 10.57 by the following:

The last remaining item is a proof of Lemma 10.56. We write

\[ S = (u_1, u_2, \ldots, u_n), \quad S_k = (u_1^{(k)}, \ldots, u_n^{(k)}) \]

in columnar form. Let \( t_{ij}^{(k)} \) denote the entries of the positive upper triangular matrix \( T_k \). The first column of the limiting equation \( S_k T_k \to S \) reads

\[ t_{11}^{(k)} u_1^{(k)} \to u_1. \]

Since both \( u_1^{(k)} \) and \( u_1 \) are unit vectors, and \( t_{11}^{(k)} > 0 \),

\[ \| t_{11}^{(k)} u_1^{(k)} \| = t_{11}^{(k)} \to \| u_1 \| = 1, \quad \text{and hence} \quad u_1^{(k)} \to u_1. \]

The second column reads

\[ t_{12}^{(k)} u_1^{(k)} + t_{22}^{(k)} u_2^{(k)} \to u_2. \]

Taking the inner product with \( u_1^{(k)} \to u_1 \) and using orthonormality, we deduce \( t_{12}^{(k)} \to 0 \), and so \( t_{22}^{(k)} u_2^{(k)} \to u_2 \), which, by the previous reasoning, implies \( t_{22}^{(k)} \to 1 \) and \( u_2^{(k)} \to u_2 \). The proof is completed by working in order through the remaining columns, employing a similar argument at each step. Details are left to the interested reader.
*** Page 591 *** Equation (11.21):

Insert minus sign before integral:

\[ u'(\ell) = - \int_{0}^{\ell} f(x) \, dx = 0, \]  

(11.21)

*** Page 594 *** Line before (11.28):

Change “to satisfy” to “satisfy”.

*** Page 598 *** 3 lines after (11.40):

Change \( L[u] = u(y) \) to \( L_y[u] = u(y) \).

*** Page 607 *** Equation (11.60):

Missing factor of \( c \) in differential equation:

\[-c u'' = f(x), \quad u(0) = 0 = u(1),\]

*** Page 607 *** Equation (11.59):

The middle expression is missing a \( c \) in the denominator:

\[ G(x, y) = \frac{(1 - y)x - \rho(x - y)}{c} = \begin{cases} x(1 - y)/c, & x \leq y, \\ y(1 - x)/c, & x \geq y, \end{cases} \]

(11.59)

*** Page 608 *** Lines 10 and 7 from bottom:

Two missing factors of \( c \):

\[ c \frac{du}{dx} = (1 - x)xf(x) + \int_{0}^{x} [-yf(y)] \, dy - x(1 - x)f(x) + \int_{x}^{1} (1 - y)f(y) \, dy = -\int_{0}^{1} yf(y) \, dy + \int_{x}^{1} f(y) \, dy. \]

Differentiating again, we conclude that \( c \frac{d^2u}{dx^2} = -f(x) \), as claimed.

*** Page 619 *** Exercise 11.3.16 (b):

Delete “is” after \( K = L^* \circ L \).

*** Page 640 *** Section 11.6, middle of second paragraph:

Change “Chapter 4.1” to “Section 4.1”.

*** Page 653 *** Solution 1.2.4 (d):

\[ A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & -1 & 3 \\ 3 & 0 & -2 \end{pmatrix}, \quad x = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}; \]
Solution 1.2.4 (f):
\[ b = \begin{pmatrix} -3 \\ -5 \\ 2 \\ 1 \end{pmatrix}. \]

Solution 1.4.15 (a):
\[
\begin{pmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{pmatrix}.
\]

Solution 1.8.4:
(i) \( a \neq b \) and \( b \neq 0 \); (ii) \( a = b \neq 0 \), or \( a = -2 \), \( b = 0 \); (iii) \( a \neq -2 \), \( b = 0 \).

Solution 1.8.23 (e):
\[(0, 0, 0)^T; \]

Solution 2.5.5 (b):
\[
x^* = (1, -1, 0)^T, \quad z = z \left( -\frac{2}{7}, -\frac{1}{7}, 1 \right)^T; \]

Solution 3.4.22 (v):
Change “null vectors” to “null directions”.

Solution 3.4.32:
Change all \( x \)'s to \( z \):
\[
0 = z^T K z = z^T A^T C A z = y^T C y, \quad \text{where} \quad y = A z. \quad \text{Since} \quad C > 0, \quad \text{this implies} \quad y = 0, \quad \text{and hence} \quad z \in \ker A = \ker K.
\]

Solution 4.4.27 (a):
Change “the interpolating polynomial” to “an interpolating polynomial”.

Solution 4.4.52 (b):
Delete the sentence:
(The solution given is for the square \( S = \{ 0 \leq x \leq 1, \ 0 \leq y \leq 1 \} \).)

Solution 5.1.14 (a):
\[
v_2 = \pm \left( -\sin \theta, \frac{1}{\sqrt{2}} \cos \theta \right)^T
\]
Solution 5.4.15:

\[ p_0(x) = 1, \quad p_1(x) = x, \quad p_2(x) = x^2 - \frac{1}{3}, \quad p_3(x) = x^3 - \frac{9}{10}x. \]

(The solution given in the text is for the interval \([0, 1]\), not \([-1, 1]\).)

Solution 5.5.6 (ii) (c):

\[ \left( \frac{23}{43}, \frac{19}{43}, -\frac{1}{43} \right)^T. \]

The page layout is a bit strange. The top of the second column (before Solution 6.2.1) is the solution to Exercise 6.1.16(c). Also, the solution to Exercise 6.2.10 spans across both columns.

Solution 6.2.1 (b) The solution corresponds to the revised exercise — see correction on page 311.

For the given matrix, the solution is

\[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

Solution 6.2.12 & 6.2.13:

Change all e’s to y’s.

Solution 6.3.5 (b):

\[ \begin{align*}
\frac{3}{2}u_1 - \frac{1}{2}v_1 - u_2 &= f_1, \\
-\frac{1}{2}u_1 + \frac{3}{2}v_1 &= g_1, \\
-u_1 + \frac{3}{2}u_2 + \frac{1}{2}v_2 &= f_2, \\
\frac{1}{2}u_2 + \frac{3}{2}v_2 &= g_2.
\end{align*} \]

Solution 8.5.1 (a):

\[ \sqrt{3} \pm \sqrt{5} \]

Solution 8.5.26:

Change (b) to (c).

Solution 9.1.28 (g):

Change \( \hat{u} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} u \) to \( \hat{u} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} u. \)

Solution 10.3.24 (e):

Change \(-2.69805 \pm .806289 \) to \(-2.69805 \pm .806289 i. \)
Solution 11.2.8 (d):

\[ f'(x) = 4 \delta(x + 2) + 4 \delta(x - 2) + \begin{cases} 
1, & |x| > 2, \\
-1, & |x| < 2,
\end{cases} \]

\[ = 4 \delta(x + 2) + 4 \delta(x - 2) + 1 - 2 \sigma(x + 2) + 2 \sigma(x - 2), \]

\[ f''(x) = 4 \delta'(x + 2) + 4 \delta'(x - 2) - 2 \delta(x + 2) + 2 \delta(x - 2). \]

Solution 11.2.31 (a):

\[ u_n(x) = \begin{cases} 
x(1 - y), & 0 \leq x \leq y - \frac{1}{n}, \\
-\frac{1}{4} n x^2 + \left( \frac{1}{2} n - 1 \right) x y - \frac{1}{4} n y^2 + \frac{1}{2} y + \frac{1}{2} x - \frac{1}{4n}, & |x - y| \leq \frac{1}{n}, \\
y(1 - x), & y + \frac{1}{n} \leq x \leq 1.
\end{cases} \]

Solution 11.3.3 (c):

(i) \[ u_*(x) = \frac{1}{2} x^2 - \frac{5}{2} + x^{-1}, \]

(ii) \[ P[u] = \int_1^2 \left[ \frac{1}{2} x^2 (u')^2 + 3 x^2 u \right] dx, \quad u'(1) = u(2) = 0, \]

(iii) \[ P[u_*] = -\frac{37}{20} = -1.85, \]

(iv) \[ P[x^2 - 2x] = -\frac{11}{6} = -1.83333, \quad P[-\sin \frac{1}{2} \pi x] = -1.84534. \]

Solution 11.5.7 (b):

\[ \lambda = -\omega^2 < 0, \quad G(x, y) = \begin{cases} 
\frac{\sinh \omega (y - 1) \sinh \omega x}{\omega \sinh \omega}, & x < y, \\
\frac{\sinh \omega (x - 1) \sinh \omega y}{\omega \sinh \omega}, & x > y;
\end{cases} \]

\[ \lambda = 0, \quad G(x, y) = \begin{cases} 
x(y - 1), & x < y, \\
y(x - 1), & x > y;
\end{cases} \]

\[ \lambda = \omega^2 \neq n^2 \pi^2 > 0, \quad G(x, y) = \begin{cases} 
\frac{\sin \omega (y - 1) \sin \omega x}{\omega \sin \omega}, & x < y, \\
\frac{\sin \omega (x - 1) \sin \omega y}{\omega \sin \omega}, & x > y.
\end{cases} \]