Introduction to Partial Differential Equations

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Corrections to Second Printing (2016)

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Exercise 1.10(a): change $4t^2 - x^2$ to $4t^2 + x^2$.

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In the last two displayed formulas, the first term on the right hand side of the equals sign is missing a minus sign:

$$\frac{\partial v}{\partial \xi}(\xi,\eta) = -\frac{1}{2c}\frac{\partial u}{\partial t}\left(\frac{\eta-\xi}{2c},\frac{\eta+\xi}{2}\right) + \frac{1}{2}\frac{\partial u}{\partial x}\left(\frac{\eta-\xi}{2c},\frac{\eta+\xi}{2}\right),$$

and so, in particular,

$$\frac{\partial v}{\partial \xi}(\xi,\xi) = -\frac{1}{2c}\frac{\partial u}{\partial t}(0,\xi) + \frac{1}{2}\frac{\partial u}{\partial x}(0,\xi) = 0,$$

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In (4.37), change final $+$ to $-$:

$$u(t,x) \approx \tfrac{1}{2}a_0 + e^{-t}(a_1\cos x + b_1 \sin x) = \tfrac{1}{2}a_0 + r_1 e^{-t}\cos(x-\delta_1), \qquad (4.37)$$

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Line -7: Change "... appear the context of boundary value problems." to "... appear in the context of boundary value problems."

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Last line of table: Change $x^4 - 4x^2y^2 + y^4$ to $x^4 - 6x^2y^2 + y^4$

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Example 5.4: change rest of sentence after displayed formula to

used earlier in Example 4.1, along with homogeneous Dirichlet boundary conditions: $u(t,0) = u(t,1) = 0$.

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Equation (5.28): change $\mathrm{O}\big((\Delta t)^2\big)$ to $\mathrm{O}(\Delta t)$:

$$\frac{\partial u}{\partial t}(t_j, x_m) \approx \frac{u(t_j, x_m) - u(t_{j-1}, x_m)}{\Delta t} + \mathrm{O}(\Delta t). \tag{5.28}$$

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Change sentence after equation (5.40):

We use step sizes $\Delta t = \Delta x = .005$, set $\ell = 1$, and try four different values of the wave speed.

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Equation (5.45): change denominator to $2\,\Delta x$:

$$\frac{\partial u}{\partial x}(t_j, x_m) \approx \frac{u_{j,m+1} - u_{j,m-1}}{2\,\Delta x} + \mathrm{O}\big((\Delta x)^2\big). \tag{5.45}$$

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Correct last displayed equation by switching indices on the $u_{i,j}$:

$$\begin{aligned} &u_{1,1} = .1831, && u_{2,1} = .2589, && u_{3,1} = .1831,\\ &u_{1,2} = .3643, && u_{2,2} = .5152, && u_{3,2} = .3643,\\ &u_{1,3} = .5409, && u_{2,3} = .7649, && u_{3,3} = .5409, \end{aligned}$$

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Equation (5.78): the sub- and super-diagonal matrix elements should be -1, not $-\rho^2$:

$$B_\rho = \begin{pmatrix} 2(1+\rho^2) & -1 & & & & & \\ -1 & 2(1+\rho^2) & -1 & & & & \\ & -1 & 2(1+\rho^2) & -1 & & & \\ & & -1 & 2(1+\rho^2) & -1 & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & -1 & 2(1+\rho^2) & -1 \\ & & & & & -1 & 2(1+\rho^2) \end{pmatrix} \tag{5.78}$$

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Equation (5.82): replace $\mathbf{w}^{(n-1)}$ by $U_{n-1}\mathbf{w}^{(n-1)}$ and U_j by U_k:

$$\begin{aligned} &\mathbf{z}^{(1)} = \widehat{\mathbf{f}}^{(1)}, && \mathbf{z}^{(j+1)} = \widehat{\mathbf{f}}^{(j+1)} - L_j \mathbf{z}^{(j)}, && j = 1, 2, \ldots, n-2,\\ &U_{n-1}\mathbf{w}^{(n-1)} = \mathbf{z}^{(n-1)}, && U_k \mathbf{w}^{(k)} = \mathbf{z}^{(k)} - \rho^2\,\mathbf{w}^{(k+1)}, && k = n-2, n-3, \ldots, 1, \end{aligned} \tag{5.82}$$

Equation (5.83): replace L_j by L_k:

$$\mathbf{w}^{(k)} = L_k\big(\mathbf{w}^{(k+1)} - \rho^{-2}\,\mathbf{z}^{(k)}\big), \qquad k = n-2, n-3, \ldots, 1. \tag{5.83}$$

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Equation (6.40): delete initial fraction:

$$\int_{-\pi}^{\pi} s_n(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\frac{1}{2}x}\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_{k=-n}^{n} e^{\,i\,k\,x}\,dx = 1, \qquad (6.40)$$

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In the first integral in the displayed equation after (6.65) change $\sinh\omega\,y$ to $\sinh\omega\,\xi$:

$$u(x) = \int_0^x \frac{\sinh\omega\,(1-x)\sinh\omega\,\xi}{\omega\sinh\omega}\,d\xi + \int_x^1 \frac{\sinh\omega\,x\,\sinh\omega\,(1-\xi)}{\omega\sinh\omega}\,d\xi$$

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Displayed formula after Theorem 6.17: change $\mathbb{R}^2$ to Ω:

$$u(x,y) = -\iint_\Omega G_0(x,y;\xi,\eta)\,\Delta u(\xi,\eta)\,d\xi\,d\eta.$$

Equation (6.108): change $\mathbb{R}^2$ to Ω twice:

$$\iint_\Omega \delta(x-\xi)\,\delta(y-\eta)\,u(\xi,\eta)\,d\xi\,d\eta = \iint_\Omega -\Delta G_0(x,y;\xi,\eta)\,u(\xi,\eta)\,d\xi\,d\eta. \qquad (6.108)$$

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Equation (6.135): correct left hand side:

$$\frac{\partial G}{\partial\rho}(r,\theta;1,\phi) = -\frac{1}{2\pi}\,\frac{1-r^2}{1+r^2-2\,r\cos(\theta-\phi)}\,, \qquad (6.135)$$

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In the displayed equation immediately above the Exercises, delete one factor of $1/k$ in the first term after the equals sign:

$$\widehat{f}(k) = \left(-\frac{i}{k}\sqrt{\frac{\pi}{2}}\,e^{-|\,k\,|} + \frac{\pi^{3/2}}{\sqrt{2}}\,\delta(k)\right) - \frac{\pi^{3/2}}{\sqrt{2}}\,\delta(k) = -\,i\sqrt{\frac{\pi}{2}}\,\frac{e^{-|\,k\,|}}{k}\,.$$

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Exercise 7.2.12. Insert factor of $\sqrt{2\pi}$ in formula $\widehat{f}(k) = \sqrt{2\pi}\sum_{n=-\infty}^{\infty} c_n\,\delta(k-n)$.

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Second displayed formula after equation (8.63): insert missing factor of $\frac{1}{2}$:

$$u(t,x) = c_1 + c_2\,\mathrm{erf}\left(\frac{x}{2\sqrt{t}}\right).$$

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Exercise 10.3.16: Change $n = 2$ in part (b) to $n = 3$, and change $n = 3$ in part (c) to $n = 4$.

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Case (iii): Change $r_2 = r_1 + k$ to $r_1 = r_2 + k$; change "smaller" to "larger", and change x^{r_2} to $(x - x_0)^{r_2}$ in equation (11.93):

(iii) Finally, if $r_1 = r_2 + k$, where $k > 0$ is a positive integer, then there is a nonzero solution $\widehat{u}(x)$ with a convergent Frobenius expansion corresponding to the larger index r_1. One can construct a second independent solution of the form

$$\widetilde{u}(x) = c \log(x - x_0)\, u^{(\infty)}(x) + v(x), \quad \text{where} \quad v(x) = (x - x_0)^{r_2} + \sum_{n=1}^{\infty} v_n (x - x_0)^{n + r_2} \tag{11.93}$$

is a convergent Frobenius series, and c is a constant, which may be 0, in which case the second solution $\widetilde{u}(x)$ is also of Frobenius form.

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In equation (12.145) and the displayed equation immediately after, the limit should be as $t \to 0$:

$$\lim_{t \to 0} \mathrm{M}_{ct}[\,f\,] = \mathrm{M}_0[\,f\,] = f(\mathbf{0}). \tag{12.145}$$

$$\lim_{t \to 0} \langle\, u(t,\cdot)\,;f\,\rangle = \langle\, u(0,\cdot)\,;f\,\rangle = 0 \qquad \text{for } \textit{all} \text{ functions} \quad f,$$

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