Corrections to the Corrected Printing and Paperback Edition of

Olver, P.J., *Applications of Lie Groups to Differential Equations*,
Second Edition, Springer–Verlag, New York, 1993.

Last updated: May 7, 2019

⋆⋆⋆ *On page 10, line 6, change*

$\phi \circ \tilde{\phi}^{-1} \colon \mathbb{R} \to \mathbb{R}$

to

$\phi^{-1} \circ \tilde{\phi} \colon \mathbb{R} \to \mathbb{R}$

⋆⋆⋆ *On page 19, line 17, change*

$x \in V_0 = \{\, x : |\, x \,| < \frac{1}{2} \,\}$

to

$x \in V_0 = \{\, -1 < x < \frac{1}{3} \,\}$

⋆⋆⋆ *On page 36, line 9, change*

$\varepsilon, \theta \in V$

to

$(\varepsilon, \theta) \in V$

⋆⋆⋆ *On page 43, line 23, change*

smoth vector fields

to

smooth vector fields

⋆⋆⋆ *On page 52, Example 1.58, change*

Lie proved that

to

Lie proved, [4], that

⋆⋆⋆ *On page 67, line 15, change*

sort

to

short

⋆⋆⋆ *On page 70, Exercise 1.8, change*

in polar coordinates,

to

in polar coordinates with $r > 0$,

$\star\star\star$ *On page 72, Exercise 1.24(b), change*

$\mathfrak{h} \subset \mathfrak{g}$ has the property

to

$\mathfrak{h} \subset \mathfrak{g}$ is a *normal subalgebra* (or *ideal*), meaning that it has the property

$\star\star\star$ *On pages 79–81, the ends of the proofs of Proposition 2.6 and Theorem 2.8 that refer back to* (1.40) *are subtly flawed, in that, under our definition of a local group action, even when $g = g_1 \cdot g_2$ with $g, g_1, g_2 \in G_x$, there is no guarantee that $g_1 \in G_{g_2 \cdot x}$, i.e., that $g_1 \cdot (g_2 \cdot x)$ is defined even though $g \cdot x = (g_1 \cdot g_2) \cdot x$ is defined.*

However, a more direct argument, that avoids this difficulty, is to replace the end of the proof of Proposition 2.6 by the following:

... Conversely, if (2.1) holds everywhere, then

$$\frac{d}{d\varepsilon}\, \zeta(\exp(\varepsilon\, \mathbf{v})\, x) = 0$$

where defined, and hence $\zeta(\exp(\varepsilon\mathbf{v})\,x) = \zeta(x) = c$ is constant for ε in the connected interval containing 0 in $\{\, \varepsilon \in \mathbb{R} \mid \exp(\varepsilon\,\mathbf{v}) \in G_x \,\}$. Using the fact that the exponential map is a local diffeomorphism from a neighborhood of $0 \in \mathfrak{g}$ to a neighborhood of $e \in G_x$, we conclude that $\zeta(g \cdot x) = c$ for all g in an open neighborhood of the identity in G_x. Now, set $\widetilde{G}_x = \{\, g \in G_x \mid \zeta(g \cdot x) = c \,\}$. Applying the preceding argument at the point $g \cdot x$ for any $g \in \widetilde{G}_x$ proves that $\widetilde{G}_x$ is open, while continuity proves that it is closed in G_x. Thus, by connectivity, $\widetilde{G}_x = G_x$, and the result follows. $\square$

Similarly, replace the end of the proof of Theorem 2.8 by:

... We have thus shown that if x_0 is a solution to $F(x) = 0$, and $\mathbf{v}$ is an infinitesimal generator of G, and ε is sufficiently small, then $\exp(\varepsilon\,\mathbf{v})\,x_0$ is also a solution. As in the proof of Proposition 2.6, one then shows that $\widetilde{G}_x = \{\, g \in G_x \mid F(g \cdot x_0) = 0 \,\}$ is both open and closed in G_x, and hence, by connectivity, $\widetilde{G}_x = G_x$. $\square$

- *Thanks to Colin James Stockdale Klaus for correspondence on this point.*

$\star\star\star$ *On page 83, line 14, change*

folowing

to

following

$\star\star\star$ *On page 93, line 8, change*

functon

to

function

⋆⋆⋆ *On page 106, line -5, the term inside the parentheses should be* $\Xi_\varepsilon(x)$. *Thus, the equation should read*

$$\sum_l \frac{\partial^2 \Xi^k_{-\varepsilon}}{\partial \tilde{x}^j \, \partial \tilde{x}^l} \big(\Xi_\varepsilon(x) \big) \frac{d\, \Xi^l_\varepsilon}{d\varepsilon}(x) = 0,$$

⋆⋆⋆ *On page 111, line 3, change*

$\mathrm{J}\tilde{f}_\varepsilon(x)$

to

$\mathrm{J}\tilde{f}_\varepsilon(\tilde{x})$

⋆⋆⋆ *On page 123, line 5, change*

$\tau \dfrac{\partial}{\partial \tau}$

to

$\tau \dfrac{\partial}{\partial t}$

⋆⋆⋆ *On page 151, line after* (2.110), *change*

normal subalgebra of $\mathfrak{g}$

to

normal subalgebra (ideal) of $\mathfrak{g}$

⋆⋆⋆ *On page 163, line -3, change*

differential equatons

to

differential equations

⋆⋆⋆ *On page 167, line 2, change*

$\binom{p+k-1}{k} \cdot l$

to

$\binom{p+k}{k} \cdot l$

⋆⋆⋆ *On page 167, Definition 2.83, change*

differential equatons

to

differential equations

⋆⋆⋆ *On page 169, line -9, change*

Exercise 2.32

to

Exercise 2.33

- *Thanks to Mariano Hermida de La Rica for the preceding four corrections.*

⋆⋆⋆ *On page 187, line 8, change*

with respect y

to

with respect to y

⋆⋆⋆ *On page 187, line 14, change*

$$\frac{\partial \delta}{\partial \hat{x}} + \frac{\partial \delta}{\partial y}\,\frac{\partial \eta}{\partial x} + \frac{\partial \delta}{\partial v}\,\frac{\partial v}{\partial y}\,\frac{\partial \eta}{\partial x}$$

to

$$\frac{\partial \delta}{\partial \hat{x}}\,\frac{\partial \hat{x}}{\partial x} + \frac{\partial \delta}{\partial y}\,\frac{\partial \eta}{\partial x} + \frac{\partial \delta}{\partial v}\,\frac{\partial v}{\partial y}\,\frac{\partial \eta}{\partial x}$$

- *Thanks to Wen-xiu Ma and his class for catching this and several other errors.*

⋆⋆⋆ *On page 202, lines 13–14, change*

linear system of ordinary differential equations,

to

linear, constant coefficient system of ordinary differential equations,

⋆⋆⋆ *On page 207, line -2, change*

Example 2.64

to

Example 2.44

⋆⋆⋆ *On page 212, on both line 12 and line 13, change*

M/G

to

M/G^2

⋆⋆⋆ *On page 214, line 13, change*

$\widetilde{F}(x) = F[\pi(x)]$

to

$F(x) = \widetilde{F}[\pi(x)]$

⋆⋆⋆ *On page 215, line -1, change*

$\widetilde{F} = \widetilde{F}(\pi_1, \ldots, \pi_{m-s})$

to

$\widetilde{F} = \widehat{F}(\pi_1, \ldots, \pi_{m-s})$

$\star\star\star$ *On page 217, line 6, change*

$F(R,K)=0$

to

$\widehat{F}(R,K)=0$

$\star\star\star$ *On page 238, Exercise 3.7, change the formula to*

$$R=\left(\frac{t^2E}{p_0}\right)^{1/5}\; h\left[p_0\left(\frac{t^6}{\rho_0^3\,E^2}\right)^{1/5}\right]$$

- *Thanks to Kameron Decker Harris for spotting this.*

$\star\star\star$ *On page 273, line -7, change*

tecnique

to

technique

$\star\star\star$ *On page 280, in the table, change*

$I_x = x\,D - y\,A + \frac{1}{2}\,x\,u\,u_t + t\,M_x$

to

$I_x = x\,D + y\,A + \frac{1}{2}\,x\,u\,u_t + t\,M_x$

- *Thanks to Gehrt Hartjen for checking through this table and the table on page 340 in his Mathematics Diplomarbeit in Aachen, 2001.*

$\star\star\star$ *On page 290, line -4, change*

Exercise 2.33

to

Exercise 2.35

$\star\star\star$ *On page 331, insert minus sign after equals sign in second displayed equation:*

$$D_\Delta^* Q = -\,q_t - q_{xx} + u\,q_x$$

$\star\star\star$ *On page 340, in line 4 of the table, change*

$-\,y\,u_{xxx} + x\,u_{xyy} + u_{xy}$

to

$-\,y\,u_{xxx} + x\,u_{xxy} + u_{xy}$

$\star\star\star$ *On page 340, in line 5 of the table, change*

$$u_{xx}(y\,u_{yt} + \tfrac{1}{2}\,u_t) - u_{yy}(x\,u_{xt} + \tfrac{1}{2}\,u_t)$$

to

$$-\,u_{xx}(y\,u_{yt} + \tfrac{1}{2}\,u_t) + u_{yy}(x\,u_{xt} + \tfrac{1}{2}\,u_t)$$

$\star\star\star$ *On page 350, lines -8 to -7, change*

the their Fréchet

to

their Fréchet

$\star\star\star$ *On page 364, Theorem 5.92, first line, change*

$P[u] \in \mathcal{A}^p$

to

$P[u] \in \mathcal{A}^q$

- *Thanks to Thomas von Schroeter for spotting this.*

$\star\star\star$ *On page 366, line -7, change*

$J \setminus I$

to

$I \setminus J$

- *Thanks to Rob Thompson for catching this and several other errors.*

$\star\star\star$ *On page 381, Exercise 3.16a*:

The system does not, in fact have a recursion operator, although there is a recursive formula for generating the higher order symmetries. On the other hand, the related system

$$u_t = u_{xx} + v^2, \qquad v_t = v_{xx},$$

does admit a recursion operator. Details can be found in: Beukers, F., Sanders, J.A., and Wang, J.P., On integrability of systems of evolution equations, *J. Diff. Eq.* **172** (2001), 396-408.

$\star\star\star$ *On page 381, Exercise 3.16b*:

A proof that the Bakirov system has only one generalized symmetry can now be found in: Beukers, F., Sanders, J.A., and Wang, J.P., One symmetry does not imply integrability, *J. Diff. Eq.* **146** (1998), 251–260.

$\star\star\star$ *On page 397, equation* (6.17)*, change closing* $\}$ *to* $\rangle$:

$$\{F,H\}(x) = \langle\, x\,;[\,\nabla F(x), \nabla H(x)\,]\,\rangle, \qquad x \in \mathfrak{g}^*, \tag{6.17}$$

⋆⋆⋆ *On page 420, line -8, change*

rocedure.

to

procedure.

⋆⋆⋆ *On page 427, lines 23–24, change*

Recently, Weinstein, [3], proposed the less historically motivated term "Casimir function" for these objects, . . .

to

Recently, R. Littlejohn, *AIP Conference Proc.* **88** (1982), 47–66, and A. Weinstein, *AIP Conference Proc.* **88** (1982), 1–11, and [3], have proposed the less historically motivated term "Casimir function" for these objects, . . .

- *Thanks to Phil Morrison for alerting me to these earlier references.*

⋆⋆⋆ *On page 469, change title of second reference by H. Bateman to*

On dissipative systems and related variational principles