Homework Assignment # 3

Exercise 3.1.
(a) Prove that the vectors
\[ v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \quad v_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \]
form an orthogonal basis of \( \mathbb{R}^3 \).
(b) Use orthogonality to write the vector \( \mathbf{x} = (1, 2, 3)^T \) as a linear combination of \( v_1, v_2, v_3 \).

Exercise 3.2.
(a) Prove that the polynomials
\[ P_0(t) = 1, \quad P_1(t) = t, \quad P_2(t) = \frac{3}{2} t^2 - \frac{1}{2}, \quad P_3(t) = \frac{5}{2} t^3 - \frac{3}{2} t, \]
form an orthogonal basis for the vector space \( \mathcal{P}^3 \) consisting of all polynomials of degree \( \leq 3 \) with respect to the \( L^2 \) inner product
\[ \langle f ; g \rangle = \int_{-1}^{1} f(t) g(t) \, dt. \]
(b) Find an orthonormal basis of \( \mathcal{P}^3 \).
(c) Write the polynomial \( t^3 \) as a linear combination of \( P_0, P_1, P_2, P_3 \) using the orthogonal basis formula (6.5).

Exercise 3.3.
(a) Determine whether the vectors
\[ v_1 = \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 1 + i \\ 2 \end{pmatrix}, \quad v_3 = \begin{pmatrix} -1 + i \\ 1 + i \\ -1 \end{pmatrix}, \]
are linearly independent or linearly dependent.
(b) Compute the Hermitian norm of each vector.
(c) Compute the Hermitian dot products between each pair of different vectors. Which vectors are orthogonal? Do the vectors form an orthogonal basis or orthonormal basis of \( \mathbb{C}^3 \)?

Due: Monday, October 1

First Midterm: Wednesday, October 10
\[ \implies \text{Will cover chapters 1–7, 10–14} \]