Homework Assignment # 6

Exercise 6.1. Find and sketch a graph of the first and second derivatives (in the context of generalized functions) of the following function:

\[ f(x) = \begin{cases} 
\sin \pi x, & x > 1, \\
1 - x^2, & -1 < x < 1, \\
e^x, & x < -1.
\end{cases} \]

Exercise 6.2. The delta function \( \delta(x) \) does not have mean zero, and so one cannot integrate its Fourier series to obtain the Fourier series for the step function \( \sigma(x) \). However, first prove that the function \( \delta(x) - \frac{1}{2\pi} \) does have mean zero. What function does the integrated Fourier series converge to? Connect this with the Fourier series for the step function.

Exercise 6.3.
(a) Prove that the derivative of the ramp function \( \rho(x) = \begin{cases} 
x, & x \geq 0, \\
0, & x \leq 0,
\end{cases} \) is the step function \( \sigma(x) \).
(b) If you differentiate the Fourier series for the ramp function you found in Homework #5, do you obtain the Fourier series for the step function? If not, explain which Fourier series you find.
(c) Find the Fourier series for the second order ramp function

\[ \rho_2(x) = \begin{cases} 
x^2, & x \geq 0, \\
0, & x \leq 0,
\end{cases} \]

by integrating your Fourier series for \( \rho(x) \). Pay careful attention to the mean.
(d) Is the derivative of your Fourier series for \( \rho_2(x) \) equal to the Fourier series for \( \rho(x) \)? Explain.

Exercise 6.4.
(a) Find the real Fourier series for the derivative of the delta function \( \delta'(x) \) directly from the definition.
(b) Compare your series with that obtained by term-by-term differentiation of the series for the delta function \( \delta(x) \).
(c) Graph the first 3, 10 and 20 terms in your Fourier series for the functions \( \delta'(x) \) and discuss what you see.

Due: Friday, November 9

Second Midterm: Monday, November 19

\[ \implies \] Will cover chapters 16–21