Exercises: Strauss pp 36–37 1, 7 (replacing odd with even), 9, p 40, 2, 3, 5, p 64 1

1. (a) Explain how to solve the Neumann initial-boundary value problem

\[ \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad \frac{\partial u}{\partial x} (t, 0) = 0 = \frac{\partial u}{\partial x} (t, 1), \quad u(0, x) = f(x), \quad \frac{\partial u}{\partial t} (0, x) = g(x), \]

on the interval \(0 \leq x \leq 1\). (b) Sketch the graph of the solution at a few representative times when

\[ f(x) = \begin{cases} 
  x - \frac{1}{4}, & 1/4 \leq x \leq 1/2, \\
  \frac{3}{4} - x, & 1/2 \leq x \leq 3/4 \\
  0, & \text{otherwise,} 
\end{cases} \]

and \( g(x) = 0 \), and discuss what is happening. Is the solution periodic in time? If so what is the period? (c) Do the same when \( f(x) = 0 \) and \( g(x) = x \).

Due: Thursday, September 30