1) \( f(x) \) is of degree 4 \( \Rightarrow \) it has exactly 4 complex zeros

- \( f(x) \) has real coefficients \( \Rightarrow \) if \( z \) is a zero, then \( \bar{z} \) is also a zero.
- \( i \) is a zero \( \Rightarrow i = -i \) is also a zero.
- \( 3+i \) is a zero \( \Rightarrow 3+i = 3-i \) is also a zero.

\( \square \)

2) \( f(x) \) is of degree 3 \( \Rightarrow \) it has exactly 3 complex zeros

- Since \( 3+i \) is a zero, \( 3-i \) is also a zero.

- \( f(x) = (x+2)(x-(3+i))(x-(3-i)) \)

\text{Trick:} \quad (x-a)(x-b) = x^2-(a+b)x+ab

\begin{align*}
a + b &= (3+i) + (3-i) = 6 \\
a b &= (3+i)(3-i) = 3^2-i^2 = 10
\end{align*}

\( f(x) = (x+2)(x^2-6x+10) \)

\( = x^3-4x^2-2x+20 \)
3) \[ f(x) = x^3 - 2x^2 - 11x + 52 \]

-4 is a zero of \( f(x) \). \( \Rightarrow \) \( f(x) \) has a factor \( x + 4 \).

Using long division, we get:

\[
\begin{array}{c|cccc}
  & 1 & -2 & -11 & 52 \\
-4 & & -4 & 24 & -52 \\
\hline
  & 1 & -6 & 13 & 0 \\
\end{array}
\]

\[ f(x) = (x + 4)(x^2 - 6x + 13) \]

- Find the zeros of \( x^2 - 6x + 13 \).

\[ \Delta = 6^2 - 4 \cdot 13 = 36 - 52 = -16 \]

- \( x^2 - 6x + 13 \) has two distinct complex zeros

\[ x = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i \]

(3)
4) \(2 + 2i\)

\[\begin{align*}
\text{Let } x &= 2, \quad y = 2 \\
r &= \sqrt{x^2 + y^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2} \\
\cos \theta &= \frac{x}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \\
\sin \theta &= \frac{y}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}
\end{align*}\]

\[\Rightarrow \theta = \frac{\pi}{4} \quad (= 45^\circ)\]

\[\Rightarrow 2 + 2i = 2\sqrt{2} \left( \cos 45^\circ + i \sin 45^\circ \right) \quad \boxed{C}\]

**Remark:** We can use

\[\tan \theta = \frac{y}{x} = \frac{2}{2} = 1\]

\[\Rightarrow \theta = 45^\circ + k \cdot 180^\circ\]

Choose \(k = 1\) because the point is in the first quadrant.