Please show all necessary work completely and clearly.

(1) (7 Points) Determine whether the function \( y(x) = xe^x \) is a solution to the initial value problem
\[
y'(x) - y(x) = e^x, \quad y(0) = 0.
\]

Answer: Yes. We have that \( y'(x) = e^x + xe^x \) and therefore \( y'(x) - y(x) = [e^x + xe^x] - xe^x = e^x \), so the differential equation is satisfied.

We also have to check the initial condition \( y(0) = 0 \). Indeed, \( y(0) = 0 \cdot e^0 = 0 \).

2 points for any work
2 points for computing \( y' \)
2 points for comparing \( y' - y \) to \( e^x \)
1 point for checking the initial condition

7 points total

(2) (3 Points) Suppose \( y(x) \) is a solution to the differential equation
\[
\frac{dy}{dx} = x^2 + xy + y^2.
\]
If the point \((1, -1)\) is on the graph of \( y(x) \), then what is \( y'(1) \)?

Answer: We have to evaluate \( \frac{dy}{dx} \) at \((x, y) = (1, -1)\). We have
\[
\frac{dy}{dx} = x^2 + xy + y^2 = 1^2 + 1(-1) + (-1)^2 = 1 - 1 + 1 = 1.
\]
1 point for any work
1 point for plugging \((1, -1)\) into \( x^2 + xy + y^2 \)
1 point for correct answer

3 points total