Quiz 2

1. At what points is $f(x)$ continuous? Explain why.

\[ f(x) = \begin{cases} 
  x + 2 & \text{if } x < 0 \\
  e^x & \text{if } 0 \leq x \leq 1 \\
  2 - x & \text{if } x > 1 
\end{cases} \]

2. Find the following limits (support your answers with calculation)

(a) \[ \lim_{x \to \infty} \frac{2x^2}{3x^2 + x + 1} \]

(b) \[ \lim_{x \to \infty} \frac{x}{x^2 + 1} \]
1. \( f(x) \) is continuous at every point in \((\infty, 0) \cup (0, 1) \cup (1, \infty)\).

Now check whether \( f(x) \) is continuous at 0 and 1.

\[
\begin{align*}
\lim_{x \to 0^-} f(x) &= \lim_{x \to 0^-} (x+2) = 0+2 = 2 \\
\lim_{x \to 0^+} f(x) &= \lim_{x \to 0^+} e^x = e^0 = 1
\end{align*}
\]

\( \lim_{x \to 0} f(x) \) DNE

\[
\begin{align*}
\lim_{x \to 1^-} f(x) &= \lim_{x \to 1^-} e^x = e^1 = e \\
\lim_{x \to 1^+} f(x) &= \lim_{x \to 1^+} (2-x) = 2-1 = 1
\end{align*}
\]

\( \lim_{x \to 1} f(x) \) DNE

Thus, \( f(x) \) is discontinuous at 0 and 1. We conclude that all the points at which \( f \) is continuous are 
\((-\infty, 0) \cup (0, 1) \cup (1, \infty)\).

2. (a) \( \lim_{x \to \infty} \frac{2x^2}{3x^2 + x + 1} = \lim_{x \to \infty} \frac{2}{3 + \frac{1}{x} + \frac{1}{x^2}} = \frac{2}{3 + 0 + 0} = \frac{2}{3} \)

(b) \( \lim_{x \to \infty} \frac{x}{x^2 + 1} = \lim_{x \to \infty} \frac{\frac{1}{x}}{1 + \frac{1}{x^2}} = \frac{0}{1 + 0} = 0 \).