1. Given a function $f(x, y) = e^{x+y} \cos(xy)$,

   (i) Compute the gradient vector $\nabla f$.

   (ii) Calculate the directional derivative $D_{\vec{a}} f$ at point $(0,0)$ in the direction of vector $\vec{a} = \langle 2, -1 \rangle$.

   (iii) Calculate the directional derivative $D_{\vec{b}} f$ at point $(0,0)$ in the direction of vector $\vec{b} = \nabla f(0,0)$. 
2. Given a function \( f : \mathbb{R}^2 \to \mathbb{R}^3 \), \( f(x, y) = (xy, x + y^2, \sin y) \),

(i) Compute the derivative matrix (or Jacobian matrix) \( Df \).

(ii) Find the derivative matrix of \( f \) at point \((1,0)\).

(iii) Calculate the linear approximation \( L(x, y) \) of \( f \) at point \((1,0)\).

(iv) Use the linear approximation above to estimate \( f(1, -0.1) \).
3. Given a function \( z = f(x, y) = x^2 - xy + y^2 \) and a point \( A(1, -1, 3) \) which lies on its graph.

(a) The cross section \( x = 1 \) of the graph is a curve. Write a direction vector of the tangent line to this curve at point \( A \).

(b) The cross section \( y = -1 \) of the graph is a curve. Write a direction vector of the tangent line to this curve at point \( A \).

(c) Write a parametric equation of the plane tangent to the graph at \( A \).

4. Let \( f \) be the same function as in Problem 3. Define \( g(x, y, z) = z - f(x, y) \).

(a) What is the level set \( g(x, y, z) = 0 \) in relation to the graph of function \( f \)?

(b) Using the principle “a level set is perpendicular to the gradient vector”, determine a normal vector of this level set at point \( A(1, -1, 3) \).

(c) What is the (cartesian) equation of the tangent plane of the graph of \( f \) at point \( A \)?
5. Let $f : \mathbb{R}^2 \to \mathbb{R}$. Put $x = ts$, $y = t + s$ and $g(t, s) = f(x, y) = f(ts, t + s)$. Express the
\[
\frac{\partial g}{\partial t}, \quad \frac{\partial g}{\partial s}, \quad \frac{\partial^2 g}{\partial t \partial s}
\]
in terms $t$, $s$ and partial derivatives of $f$. 