Note: This homework is pretty long, so it is a good idea to start early. As before, turn in only the starred problems.

Problems 19, 20, 24*, 25*, 26*, 28*, 29, 30*, 34, 38 (a), (b), (c) in Chapter I in Lang.

Additional problems:

1*) Let $G$ be a subgroup of $S_n$.
   (i) If $G \cap A_n = \{id\}$, then $|G| \leq 2$.
   (ii) If $|G| > 2$ and $G$ is simple, then $G \subset A_n$.
   (iii) If $n \geq 5$, then $S_n$ has no subgroup of index $m$ with $2 < m < n$.
   (iv) If $n \geq 5$, then $A_n$ has no subgroup of index $m$ with $2 \leq m < n$.

2*) Prove that there are no simple groups of order 90.

3*) Show that every group of order 231 is the direct product of a group of order 11 and a group of order 21.

4*) Give an example of a finite group $G$ having $p$-Sylow subgroups $P, Q$, and $R$ (for some prime $p$) with $P \cap Q = \{e\}$ and $P \cap R \neq \{e\}$. 