Turn in the starred problems only.

1*. Let $A$ be a factorial ring and $a \in A$, $a \neq 0$. Prove that the ring $A[X]/(aX - 1)$ is factorial.

2*. Let $k$ be a field.
   
   (a) Show that the ring $A := k[X, Y]/(Y^2 - X^2 - X^3)$ is an integral domain whose field of fractions is isomorphic to $k(T)$, and which is not factorial.
   
   (b) Same problem for $k[X, Y]/(Y^2 - X^3)$.

Problems 5*, 8*, 9*, 10*, 11, 17, 18* in Chapter IV in Lang.