(i) An exercise in abstraction:

(a) Show that $Y = C^0([0, 1])$ is a Banach space when equipped with the norm

$$\|u\|_{C^0} = \sup_{t \in [0,1]} |u(t)|.$$  

(b) Show that $X = C^1([0, 1]) \cap \{ u \mid u(0) = 0 \}$ is a Banach space when equipped with the norm

$$\|u\|_{C^1} = \sup_{t \in [0,1]} (|u'(t)| + |u(t)|).$$

(c) Show that the operator

$$L : X \to Y, \quad u \mapsto u',$$

is linear and bounded.

(d) Show that $L$ is bounded invertible and describe the inverse.

(ii) Find the solution to $x' = Ax \in \mathbb{R}^2$, $A = \begin{pmatrix} 1 & \mu \\ 0 & 1 \end{pmatrix}$, with $x(0) = (0,1)^T$, for any $\mu \in \mathbb{R}$.  
