(1) Consider the initial-value problem $\dot{u} = u(1-u), u(0) = u_0$.

   (i) Find the explicit solution $u(t; u_0)$ for $u_0 \in (0, 1)$ and for $u_0 = 1$. Differentiate the explicit solution with respect to $t$ and with respect to $u_0$.

   (ii) Find an initial-value problem for $\partial_{u_0} u(t; u_0)$ and for $\partial_t u(t; u_0)$ and find the explicit solutions of the initial-value problem. Compare your solution with the direct calculation from (i).

(2) Consider $\dot{x} = f(x), x(0) = x_0$ and assume that $f(x) \cdot x \leq 0$ for all $x$ with $|x|$ sufficiently large. Show that solutions exist for all $t \geq 0$.

(3) Which of the following initial value problems possess solutions for all $t \in \mathbb{R}$ — explain why!

   (i) $\dot{x} = \sin(x), x(0) = x_0$;

   (ii) $\dot{x} = x - x^3, x(0) = 2$;

   (iii) $\dot{x} = x - x^3, x(0) = 1/2$;

   (iv) $\dot{x} = \frac{4x^3 + y}{1 + x^2 + y^2}, \dot{y} = \frac{y^5 - x^5}{x^4 + y^4 + 1}$;

*Homework is due on Friday, October 10, in class*