Problems 5.31, 9.36, 9.45, 12.9. In Problem 9.36, prove not what is asked but that
\[ \lim_{n \to \infty} \frac{S_n}{M_n} = P(Y \in B) \text{ (a.s.)} \]

Additional problems:

A) Let \( X_1, X_2, \ldots \) be a sequence of nonegative pairwise nonpositively correlated random variables such that \( \mu_n = EX_n < \infty \) and \( \text{Var}X_n \leq M\mu_n \) for all \( n \), where \( M \) is a constant. By generalizing an argument given in class, prove that
\[ \sum_{n=1}^{\infty} \mu_n = \infty \Rightarrow \sum_{n=1}^{\infty} X_n = \infty \text{ (a.s.)} \]

B) Let \((E, \mathcal{F}, \mu)\) be a measure space with \( \sigma \)-finite measure. Let \( f(t, x) \) be a function on \([a, b] \times E\) measurable with respect to \( \mathcal{B}([a, b]) \otimes \mathcal{F}\). Assume that there exists a measurable function \( g(t, x) \) on \([a, b] \times E\) such that for any \( x \) we have \( f'(t, x) = f(t, x) \) in the sense explained in class. Finally, assume that
\[ \int_E \int_a^b |f'(t, x)| dt \mu(dx) < \infty, \quad \int_E |f(a, x)| \mu(dx) < \infty. \]

Prove that
\[ \left( \int_E f(t, x) \mu(dx) \right)' = \int_E f'(t, x) \mu(dx). \]

C) (Polya’s theorem) Let \( F_n, n = 1, 2, \ldots \) be a sequence of distribution functions and let \( F \) be a distribution function. Let \( \rho \) be a countable dense subset of \( \mathbb{R} \) containing all points of discontinuity of \( F \). Assume that for any \( x \in \rho \)
\[ \lim_{n \to \infty} F_n(x) = F(x), \]

and for any point \( x \) of discontinuity of \( F \)
\[ \lim_{n \to \infty} F_n(x-) = F(x-). \]

Then
\[ \lim_{n \to \infty} \sup_x |F_n(x) - F(x)| = 0. \]